PRACTICAL EXERCISES IN GEOMETRY

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PRACTICAL EXERCISES IN GEOMETRY

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PRACTICAL EXERCISES

IN

GEOMETRY

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BY

W. D. EGGAR, M.A.

ASSISTANT MASTER AT ETON COLLEGE

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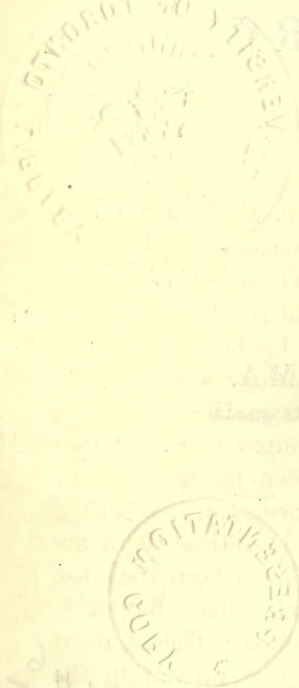

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PREFACE.

THIS book is an attempt to adapt the experimental method to the teaching of Geometry in schools. The main object of this method, sometimes called "heuristic," is to make the student think for himself, to give him something to do with his hands for which the brain must be called in as a fellow-worker. The plan has been tried with success in the laboratory, and it seems to be equally well-suited to the Mathematical class-room. Care has been taken to select exercises which can be easily performed in an ordinary room with ordinary materials. Doubtless there are many experiments, which might have been included, missing from the following collection; but it is hoped that the course will be found sufficient, or at least that it will indicate a way of presenting the work to a beginner.

It is impossible to make the earlier exercises too easy, or the instructions too explicit; but the feeling of helplessness, which besets so many when they are asked to carry out printed instructions, disappears in time, and the student gains confidence as he goes on. The advantages of the social method of the class-

room, where "iron sharpeneth iron" in rapid question and answer, are not to be lost sight of; but black-board demonstrations should in every case *follow*, rather than precede, the attempts made by the students themselves.

The present course requires of beginners the ability to read easily, to write legibly, and to cipher in decimals. The acquirement of geometrical notions and of mechanical skill at the same time is a double difficulty; the earlier chapters are therefore devoted chiefly to measurement of lengths, angles, areas, and volumes, constructions proper being postponed till Chapter X.

The greatest importance must be attached to accuracy of measurement. It is perfectly easy for a young pupil to measure a length to the nearest hundredth of an inch, or an angle to the nearest degree, and this standard should be insisted on from the very beginning. Without this exactness the experimental method becomes almost valueless. In the later chapters the order followed is one which the author has found useful in teaching Geometrical Drawing to students whose knowledge of Euclid did not extend beyond Book I. When a boy is acquiring the arithmetical notion of proportion and is beginning Algebra, the substance of Euclid, Books VI. and II. falls naturally into line with these other subjects; and at this stage Chapters XXV. to the end of the book are intended to fit in with the teaching of Algebra and Arithmetic. The methods of Chapter XXVIII. are not to be regarded as proofs of the

various mensuration formulæ, but merely as giving the beginner reasons for what must otherwise be meaningless to him.

The additional exercises at the end of some of the chapters will, it is hoped, be found useful. They are of varying difficulty, some harder questions being included for the benefit of students whose progress is rapid. A large number are followed by the letter (K). For these, the author is indebted to Mr. F. E. Kitchener and to Messrs. Macmillan for their kind permission to make use of Kitchener's "Geometrical Note-book," a work first published in 1868, but not so widely used as it deserves to be.

The author's thanks are due to Prof. R. A. Gregory, to whose suggestion the book owes it origin, and to Mr. A. T. Simmons, for their valuable assistance and advice, and for various experiments and illustrations taken from their "Exercises in Practical Physics"; to his colleagues, Mr. C. H. Allcock and Mr. J. M. Dyer; to his former colleague, Mr. S. A. Saunder of Wellington College; to Mr. C. Godfrey of Winchester College; and to Mr. A. W. Siddons of Harrow School, for many useful corrections and additions; and to Prof. G. M. Minchin, F.R.S., and Prof. Alfred Lodge, both of Cooper's Hill, whose careful reading of the proof sheets has been of the greatest assistance,

W. D. E.

Eton, *January*, 1903.

INSTRUMENTS AND APPARATUS.

A ruler graduated in inches and tenths, and in centimetres and millimetres.

Two set squares (45° and 60°). (The 60° set square should have a mark stamped on the longest edge. It can then be used like marquoise scales, the slope being 1 in 2.)

Hard pencil, *e.g.* HH.

Protractor.

Pencil compasses.

Dividers with screw adjustment.

(If dividers are used at all, it is essential that they should have a fine adjustment screw. A cheap form can be obtained from Messrs. Aston and Mander, 61 Old Compton Street, Soho, who also supply the above instruments in suitable boxes.)

Scissors.

Tracing paper.

Squared paper.

For Occasional Use.

String or thread, models of various solid figures, and a number of inch or centimetre cubes will be required. (These may be procured from any dealers in physical apparatus.)

It is recommended that the work be done in notebooks, which should be kept for reference; and that the student should make for himself separate collections of theorems and constructions. It is intended that Chapters I.-XII. should precede any course of strictly demonstrative geometry.

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PRACTICAL

EXERCISES IN GEOMETRY.

CHAPTER I.

MEASUREMENT OF LENGTH.

1. Measure in thumb-breadths the length of the paper of this page. Then measure the same length in inches, using that edge of your rule which is graduated in inches.

2. Is your thumb an inch in breadth? What is its exact breadth? Find this in two ways: (i) by direct measurement, (ii) by comparing your results in Ex. 1. (All measurements should be written down at once, with a few words to say what they are, thus:

length of paper = thumb-breadths,
" " inches.)

3. If you buy 100 yards of rope at a London shop and another 100 yards at an Edinburgh shop, ought the two lengths to be the same?

4. Suppose a dishonest shopkeeper used a yard measure which was less than a yard, and that if people complained he said his scale was correct and

theirs was wrong. How would it be decided which was right?

5. At the Standards Office of the Board of Trade, London, a bronze bar having two marks upon it a certain distance apart is carefully preserved. The distance between these two marks is called an **Imperial Standard Yard.** Why is it necessary for such a standard to be kept?

6. At one time the length of a foot varied in different parts of Great Britain, but it is now the same wherever it is measured. How do you account for this?

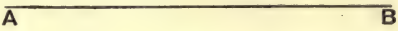


FIG. 1.

7. Measure the length of the line AB in inches. If it is not an exact number of inches, count the number of tenths of an inch in what remains over. Write your result in the **decimal system**, with a dot after the inches.

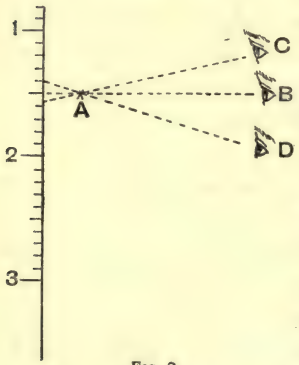


FIG. 2.

8. How many tenths of an inch are there in the same line AB (Fig. 1)?

9. In making accurate measurements, it is necessary that your measure should be close up to the line that you are measuring. For if the point A (Fig. 2) is not close up to the scale, your

reading will be correct if your eye is at *B*, but incorrect if your eye is at *C* or *D*.

You will find it convenient to use your "dividers" in measuring distances. Use them to measure the line *XY*; first put them so that the two sharp points are

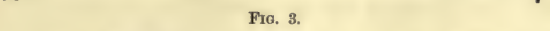


FIG. 3.

exactly at *X* and *Y*,* and then, keeping the legs of the dividers at the same angle, place the points against the scale, and read off the distance between them.

10. Measure the length of this page in inches and tenths of an inch.

11. Write the following numbers down in figures with a dot to separate the tenths from the whole numbers:

One inch and four-tenths =
Two inches and six-tenths =
Eight inches and no tenths =
No inches and five-tenths =

12. Measure the distances between the dots *AB*, *BC*, and *CD* below, in inches and tenths.

A *B* *C* *D*

Length *AB* =
„ *BC* =
„ *CD* =

Total

* If dividers are used, they should have a fine adjustment screw, and the points should always be carefully looked after.

Now measure the distance AD, and see whether it agrees with the total you have found by addition.

Length $AD =$

13. In a similar way find the total length of the following lines:

A ——— B

C ——————— D

E ——— F

G ——————— H

14. Measure the lengths of the lines AB, CD, EF, and subtract the sum of the lengths $AB+EF$ from the length CD.

A ——————— B

C ——————————— D

E ——————— F

15. You will have noticed that it is possible to measure nearer than a tenth of an inch. In Fig. 4 the inches are divided into tenths, and each tenth into

FIG. 4.

ten parts. What fraction of an inch is each of these smallest parts?

16. Write down the distance between the two marks X—X in Fig. 4 in inches, tenths, and hundredths, with a dot to separate inches from tenths.

Write the same distance in tenths.

Write the same distance in hundredths.

17. A line is 347 hundredths of an inch long. Express its length in inches and decimals of an inch.

18. With a very little practice, you will be able to guess hundredths of an inch, even if the tenths are not subdivided. It is easy to see when a point is halfway between two divisions, and not very hard to judge a third or two-thirds of the same space. Now $\frac{1}{2}=0{\cdot}5$, and $\frac{1}{3}$ and $\frac{2}{3}$ are near to 0·3 and 0·7 respectively. Thus, in Fig. 5, B is about one-third of the

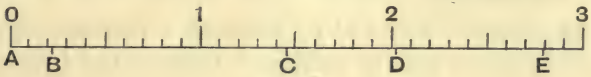


FIG. 5.

way between 0·2 and 0·3. Therefore the distance AB may be guessed as 0·23 inches. Write down (to two places of decimals) the distances AC, AD, AE.

19. Write down the following numbers in inches and decimals of an inch: 25 tenths, 462 hundredths, 220 tenths, 3708 thousandths, 150 tenths.

20. Express the following measurements in decimals of an inch: 9 tenths, 87 hundredths, 421 thousandths, 3 thousandths, 9 hundredths.

21. How many tenths of an inch are there in each of the following: 20 hundredths, 200 hundredths, 100 thousandths, 200 thousandths?

FIG. 6.

22. In Fig. 6 the line AB is divided into tenths above and into quarters and eighths below. AC is

one quarter ($\frac{1}{4}$) of the whole line. Notice that C comes half-way between two tenths and three tenths, *i.e.* two tenths five hundredths,

$$\therefore \frac{1}{4} = 0{\cdot}25.$$

Write down, as nearly as you can, the value in tenths and hundreds of the other fractions, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$. To how many places of decimals are your results correct? (You can see by turning the fractions into decimals.)

23. Suppose you had to mark off a distance equal to one-seventh part of an inch. Turning the fraction

	inch	tenths	hundredths	thousandths		
7	1·0	0	0	0	0	
	0·1	4	2	8		

into a decimal, we see that $\frac{1}{7} = 0{\cdot}1428\ldots$, and as it is with difficulty that we measure even to a hundredth of an inch, it is not of much use to go to more than three places in our calculation.

It is plain that 0·1428 is nearer to 0·1430 than to 0·1420. So we take 0·143, and as that is nearer to 0·140 than to 0·150, we take 0·14 inch as equal to $\frac{1}{7}$ of an inch as nearly as we can measure. What is $\frac{3}{7}$ inch, correct to two places?

24. Rule a straight line rather more than 3 inches long; then with your dividers take a distance as nearly as possible $\frac{2}{7}$ inch. Step this distance along your straight line seven times, being careful not to

step off the line, not to let the points slip, and not to dig deep holes in the paper. Now measure the whole distance of seven steps. It should be very nearly 3 inches.

25. Measure, to the nearest hundredth of an inch, *AB, BC, CD.*

Add these lengths together.

Measure *AD.*

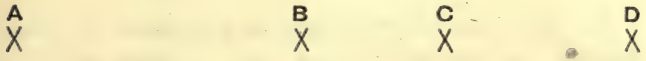


FIG. 7.

CHAPTER II.

THE METRIC SYSTEM.

1. One edge of your rule is graduated in centimetres. A hundred of these make a metre, which is the French measure corresponding to our yard, although it is rather longer than a yard. You will see that each centimetre is divided into 10 parts. These parts are called millimetres. How many millimetres are there in 1 metre?

2. Can you explain the meaning of the words centimetre and millimetre? What name should you think is given to the tenth part of a metre?

3. Copy this table, writing out in full the words that are abbreviated below:

1 m. = 10 dm.
1 dm. = 10 cm.
1 cm. = 10 mm.

4. In naming the subdivisions and multiples of the metre, the prefixes used are: *milli* ($\frac{1}{1000}$), *centi* ($\frac{1}{100}$), *deci* ($\frac{1}{10}$), *deka* or *deca* (10), *hekto* or *hecto* (100), and *kilo* (1000). Write down the names of the six units of measurement of length in which these prefixes are used, and place against each the fraction or multiple of a metre which it represents.

5. Notice on your scale how many millimetres there are in 3·5 cm., 7·6 cm., 1 dm., 11·25 cm.

6. Observe each of the following numbers on your metre scale, and write down the equivalent measure required:

225 millimetres to centimetres.
52·5 „ „ „
10·6 centimetres to millimetres.
15·55 „ „ „
2·3 decimetres „ „
2·4 „ „ centimetres.
150 millimetres to decimetres.

7. You should be able to see now that a distance of

m.	dm.	cm.	mm.
3	5	7	9

can be written thus, 3·579 metres,

or 3579 millimetres.

Write the same distance in centimetres.

8. Find how many millimetres there are in the length of this page.

Write down the result in (1) millimetres, (2) centimetres and tenths of a centimetre, (3) decimetres and tenths and hundreds of a decimetre.

9. Express the following distances in metres and decimals of a metre:

5 m. 3 dm. 9 cm.
7 dm.
2 dm. 7 mm.
5 cm.
9 mm.

10. Express the distances in Ex. 9 as centimetres and decimals of a centimetre.

11. You have now to find out the number of centimetres in 1 inch. Do this first by taking a distance of 1 inch with your dividers and then measuring it on the centimetre scale. State your result thus:

$$1 \text{ inch} = \quad \text{cm.} = \quad \text{mm.}$$

12. You can find the number of centimetres in 1 inch more accurately by taking a larger distance and measuring it in inches and then in centimetres. Thus, a line is measured and found to be 3·75 inches long. The same line is measured in cm. and found to be 9·5 cm. long. Hence

$$3{\cdot}75 \text{ inches} = 9{\cdot}5 \text{ cm.}$$

$$\therefore \ 1 \text{ inch} = \frac{9{\cdot}5}{3{\cdot}75} \text{ cm.} = \frac{950}{375} \text{ cm.} = 2{\cdot}53\ldots \text{ cm.}$$

Now this result is not quite accurate in the second place of decimals. See if you can get a better result by measuring the line AB in inches, and in centimetres.

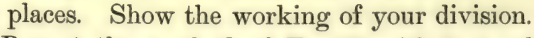


FIG. 8.

Remember that if your result is to be correct to two places, your measurements must also be correct to two places. Show the working of your division.

13. Repeat the method of Ex. 12 with two other distances, the length and breadth of your sheet of paper for instance. It does not matter what length is chosen provided that you measure it correctly both in inches and in centimetres and work the division

correct to two places. If all your measurements have been done with equal care, you can take the average of your three results by adding them all together and dividing by 3. Enter your results thus:

1 inch = cm. (i)
„ = cm. (ii)
„ = cm. (iii)

3 ⟌ ____

Average = cm.

To how many places is it necessary to take your division?

Another way of taking the average is by adding all the lengths together, both in inches and in centimetres, and then doing the division.

14. Find, by examining your rule, the equivalent, in metric measure, of four inches. Find also the number of centimetres in $7\frac{1}{2}$ inches, and in 11 inches.

15. Measure the length of the paper of this page both in inches and centimetres; also determine other lengths in the two systems of measurement.

Put down the results in parallel columns, as shown below, and from them calculate the number of centimetres in an inch.

Length in centimetres.	Length in inches.	$\frac{\text{No. of centimetres}}{\text{No. of inches}}$.

Take the average of your results in the third column.

16. Measure the length of your desk, or of a table, or other convenient object, in inches and decimals (tenths) of an inch. Determine the same length in metres and decimals of a metre. Use the results to determine the number of inches in a metre.

ADDITIONAL EXERCISES.

II. a.

1. The distance 2 metres 3 dm. 5 cm. 8 mm. can be written thus, 2·358 metres. Write in the same way the following distances :—

3 metres 1 dm. 4 cm. 7 mm.
2 metres 5 cm. 8 mm.
2 centimetres.
5 millimetres.

Add up the four distances and express the sum in millimetres.

2. Draw a line 75 millimetres in length ; write down its length in inches, and thence calculate the number of inches in a metre.

3. Draw on your paper two straight lines, and mark off on them the bore of the 4·7 in. Naval gun and of the 15 cm. Creusot gun. About what bore would the latter be in English measure? Give a rough rule for turning centimetres into inches.

4. Draw a straight line and measure off a distance of 1 dm. 2 cm. 7 mm. How many millimetres does it contain ?

5. Measure along the edge of the table a distance of half a metre. Find, by measurement, how many inches it contains, to the nearest tenth.

6. How many pieces, 43 metres long, can be cut from a wire 5329· 6 metres long, and how many centimetres will remain over ?

7. How many centimetres are there in 437 kilometres 5 metres?

8. Given that a metre is 39·37079 inches, show that the difference between 5 miles and 8 kilometres is nearly 51 yards (a kilometre is 1000 metres).

9. A motor-car travels for the first 20 minutes at the rate of 6 kilometres 126 metres per hour, then, for 2 hours 20 minutes, at the rate of 30 kilometres per hour, and, afterwards, for 25 minutes, at the rate of 7 kilometres 152 metres per hour; find the total distance traversed by the car.

10. Express 0·0583 kilometres in metres and centimetres.

CHAPTER III.

STRAIGHT LINES.

1. Take a piece of string about two inches long, and hold each end between a finger and thumb (Fig. 9). Now keep the string tight, and roll the ends of it between your fingers and thumbs, and watch the

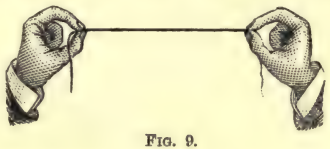


FIG. 9.

middle of the string. Does it move up and down? Now let the string be slack, and again roll the ends of it quickly. Do you notice any difference? Draw two pictures showing the difference. Can you tell when the string is straight?

2. Put a piece of tracing-paper over the line AB

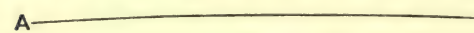


FIG. 10.

and make a careful tracing of it. Mind that the paper does not slip. Mark A and B. Now turn the tracing-paper over face downwards, and put it so that the reversed A comes on A and the reversed B on B. Does your traced line lie along AB all the way; in other

words, do the two lines **coincide**? Keep your tracing for inspection.

3. Make a tracing of the line XY in the same way.

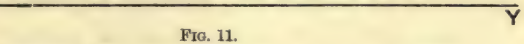


FIG. 11.

When you reverse it, does it coincide with XY?

4. You can now tell whether a line is **straight** or not. Test your ruler, to see if the lines you rule with it are straight. Do this first with tracing paper.

5. Can you invent another way of testing the straightness of the lines without using anything but the rule itself or by eye alone?

6. Your lines should all be drawn as fine as possible. Strictly speaking, a line has no breadth. For instance the edge of this paper and the boundary between the black and the white in Fig. 12 are **lines**. But in drawing black lines on white paper we are obliged to give them a certain breadth in order to see them. It is a safe rule to make your lines always as narrow and uniform as possible.

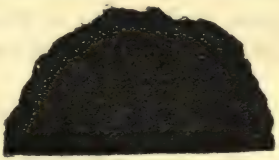


FIG. 12.

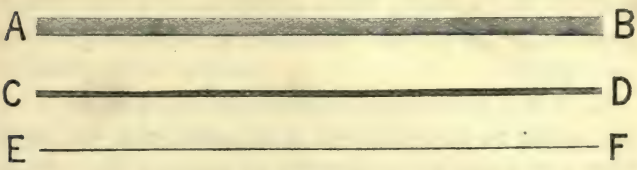


FIG. 13.

Thus in Fig. 13 EF is a better line than CD, and CD better than AB.

Sharpen your pencil so as to get a chisel edge rather than a point, and put this edge **along** the rule when you rule a line. Now rule six straight lines of any length as narrow and uniform as possible.

Where two of our lines cross one another there is a small space (blackened in Fig. 14) which belongs

FIG. 14.

to both lines. The narrower the lines the smaller is this space, until, when the lines are perfect lines, it becomes a **point.** Rule two straight lines crossing, so as to give as exact a point as possible.

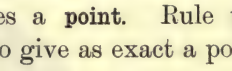
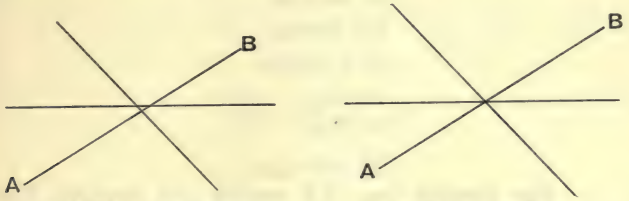


FIG. 15.

7. Rule two straight lines crossing, and then rule a third line crossing them at the same point. How many other lines could be drawn going through that same point?

8. When you are given a point, and you have to rule a line to pass through that point, you will find that you must allow for the breadth of the pencil (Fig. 16). Always try the pencil over the point, shifting the rule until you are certain that the pencil will pass through the point before you actually draw

the line. Now take two different points A and B (see Fig. 17), and rule a straight line to pass through



(i) A bad shot for the point. (ii) A good shot for the point.

FIG. 16.

both of these points. How many straight lines can be drawn from A to B? How many crooked lines?

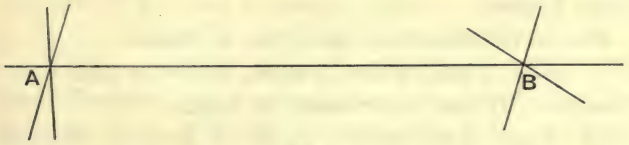


FIG. 17.

9. You have now to rule a straight line exactly 3 inches long. To do this, take your dividers and pull the legs out so that the points (which must be used with great care) are as nearly as possible 3 inches apart; then turn the adjusting screw till the distance is exactly three inches.

Now keeping the divider-points at this distance, prick two small marks in the paper so as to be just visible, and rule a straight line between the two marks. Remember that it is not necessary to dig holes. No mark on a sheet of paper should ever penetrate to the surface below.

10. In the same way rule straight lines of the following lengths, writing the length against each:

$3\frac{1}{2}$ inches.
2·6 inches.
3·75 inches.
7·5 cm.
6·95 cm.
83 mm.

11. **The straight line** AB means the straight line the ends of which are the points A and B. Draw a straight line across your paper, and mark off on it $AB = 2\frac{1}{2}$ inches, $BC = 0{\cdot}67$ inches, $CD = 1\frac{1}{4}$ inches. Then with your dividers measure the length AD, and write it down.

12. In the same way as in Ex. 11, make

$EF = 5$ cms., $FG = 23$ mms., $GH = 3{\cdot}4$ cms.

Measure EH and write down its length.

13. Place four points A, B, C, D in a straight line by eye. Then by measurement find whether $AB + BC + CD = AD$ or not.

14. Take three points A, B, C not in a straight line. Measure AC, also $AB + BC$. You will find that $AB + BC$ is always greater than AC. Try this in four different ways. From experiments of this kind you will find that **the shortest distance between two points is a straight line.**

CHAPTER IV.

THE CIRCLE.

1. Using your inch rule, set your divider points 2 inches apart. Now keep one point fixed on the paper at *A*. In how many positions can the other point be placed? Mark four of these positions, and put letters *B*, *C*, *D*, *E* against them.

2. If in place of one leg of the dividers there is an arrangement for holding a pencil point, the instrument becomes **a pair of compasses**. Set your compasses so that the pencil point is $2\frac{1}{2}$ inches from the metal point, and keep the metal point fixed on the paper, holding that leg with your left finger and thumb about one inch above the point. With your right hand take hold of the joint of the compasses and keep them upright. You can thus make your pencil pass through all the points on the paper that are $2\frac{1}{2}$ inches from the fixed point without any fear of slipping. The curve traced out by the pencil is called a **circle**; the fixed point is called the **centre** of the circle. Points on the curve itself are said to be on the **circumference** of the circle. Draw five straight lines from the centre to the circumference of your circle. What is the length of each of them?

2. The straight line drawn from the centre to the circumference of a circle is called a **radius**. Draw a circle of radius 2·3 inches. Rule a straight line passing through the centre, and terminated both ways by the circumference. Measure its length. This line is called a **diameter** of the circle.

4. Draw (or **describe**) a circle of 3 inches diameter.

5. A straight line joining any two points on the circumference of a circle is called a **chord**. Draw a circle of radius 2 inches, and, if you can, place in it chords 3 in., 4 in., 5 in. long. You may use your compasses for taking off the lengths of the chords. A chord passing through the centre has a special name. What is this?

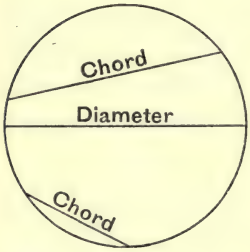


FIG. 18.

6. What is the longest chord that can be placed in a circle of radius 2·7 in.?

7. Can you think of another way of describing a circle of a given radius without using compasses?

8. A map of the London district is drawn on a scale of 1 inch to the mile. Draw the boundary line of the 4 mile cab limit (a distance of 4 miles from Charing Cross) to suit this map.

9. A circle may be called the **locus** (or path) of a point which moves so as always to be the same distance from a fixed point. A horse is tethered to a post in a field, and moves round, keeping the rope tight. What is his locus? How will it be altered if

the rope begins to coil itself round the post? Draw figures to illustrate your answers.

10. Draw a circle of 3 inches radius, and try to measure the length of its circumference with a piece of thread, or in any other way you like. Describe the method you choose, and give the result of your measurement in inches and tenths.

11. Wrap a strip of paper tightly round a wooden cylinder. (A round pencil will do if nothing larger is

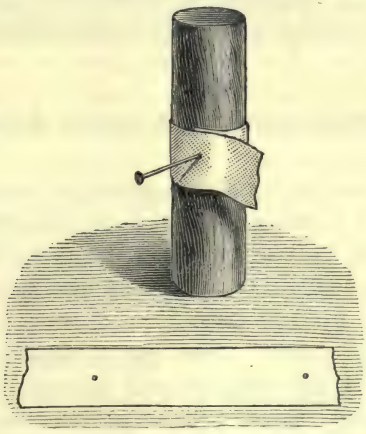


FIG. 19.

available. Bottles, ink-pots, legs of chairs are frequently cylindrical.) Then prick a hole in the paper where one fold overlaps the other (Fig. 19). When the paper is unfolded, the distance between the two holes made by the pin-point is the circumference of the circle which forms the base of the cylinder. Measure this distance as accurately as possible. Also

measure the diameter of the cylinder as accurately as possible. Your set squares will help you to do this (Fig. 20).

Divide the circumference of the cylinder by its diameter to two places of decimals

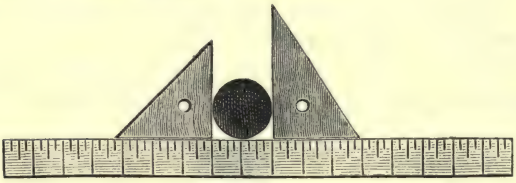


FIG. 20.

(The following exercises are suitable for out-of-school work.)

12. Try rolling a coin on paper in a straight line. If you hold it while rolling it, it will probably slip. With a little practice you can get it to roll by itself in a straight line. Or you may use the finger and thumb as a kind of axle bearing, and roll the coin between two rulers. Now put a small spot of ink on the rim of the coin, and roll the coin in a straight line as before. There will be at least two ink marks on the paper, and the distance between any mark and the next is the length of the coin's circumference, if the coin has not slipped. Measure this distance carefully. Take the average of three trials. Now measure the coin's diameter. How many times does the circumference contain the diameter? your result should be correct to the first place of decimals. Repeat this, using a different coin.

13. In Ex. 12, what result do you get from dividing the length of the circumference by that of the diameter?

14. Wind a fine string, or thread, round a lead pencil, making each turn lie close to the next. Count the number of complete turns from the beginning of the thread, or a knot in the thread. Measure the length of the thread and divide by the number of turns. This gives the circumference of the pencil. Measure the diameter of the pencil and divide length of circumference by length of diameter.

15. Procure a tracing wheel, such as is used by dressmakers. Or, obtain a small toothed wheel with sharp teeth from a disused watch or clock, and mount it with a thin nail as an axle upon a strip of wood. Run the wheel across a page of your note-book, and measure the distance in millimetres between the first and twenty-first impressions of the teeth. By dividing the result by twenty you can find the distance between two successive teeth. Repeat the determination. Write in your note-book:

Distance between 1*st* and 21*st* impressions - mm.

Therefore, distance between successive teeth, mm.

Now determine the length of a curved line by running the tracing wheel along the line and counting the number of spaces between the impressions it makes. This, multiplied by the distance between two teeth, gives the length of the curve.

CHAPTER V.

ANGLES.

1. The two hands of a watch are together at 12 o'clock. They are both pointing in the same direction; their directions **coincide**. But they do not stay together. One hand turns faster than the other, and the **angle** between the hands increases.

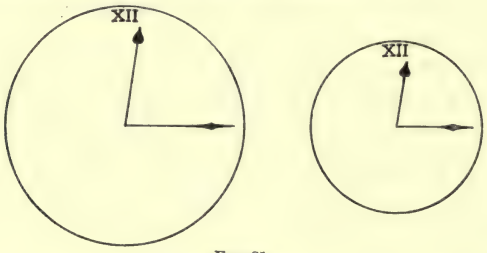


FIG. 21.

In Fig. 21 two watches of different sizes are shown recording the same time. The angle between the hands is the same in both, and it is possible to judge the time by the angle, even though the hours are not all marked. What is the time in Fig. 21?

2. What times are indicated in Fig. 22?

3. How are angles measured? They might be measured by dividing up a circle into sixty parts like the minutes on the face of a watch. But this is not

the usual way. The easiest angle to make, and to measure by, is the **right angle**. Take a piece of paper

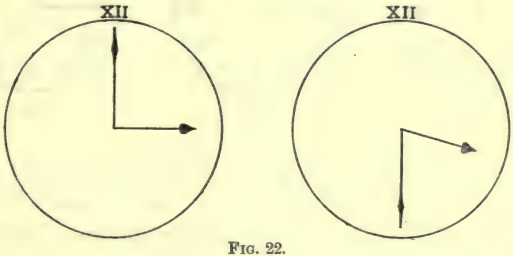


FIG. 22.

with a straight edge to it (*ABC*, Fig. 23) and fold it carefully so that *BA* comes exactly on *BC*. Then

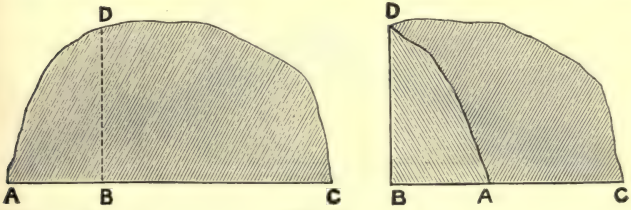


FIG. 23.

the line of the fold *BD* makes equal angles with the edge, and these angles are **right angles**.

4. Set your compasses ready for describing a circle, but turn them through one right angle only. How much of the circle have you described?

Suppose the hands of a watch are at XII (Fig. 24. (i)). I wish to put the long hand at IX. To do this I can put it back a quarter of an hour (1 right angle, Fig. 24 (ii)), or I can put it on three-quarters of an hour (Fig. 24 (iii), in which case I must turn it through 3 right angles.

FIG. 24.

The times shown are different because the short hand has moved also. But if there were no short hand it would be impossible to tell whether the long hand had

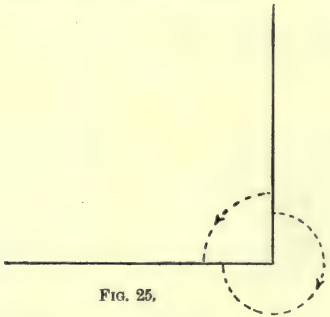


FIG. 25.

been turned back or forward. In Fig. 25 the dotted arcs are the only indication. One arc shows 1 right

FIG. 26.

angle, the other shows 3 right angles. If there is nothing to show which angle is meant, as in Fig. 26,

we take the angle which requires the least amount

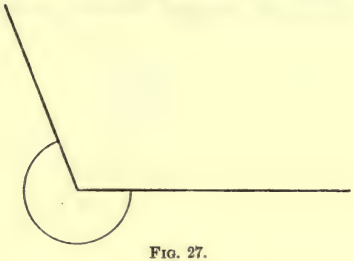


FIG. 27.

of turning. The other angle is called a **reflex angle** and needs to be shown by an arc (Fig. 27). Reflex =

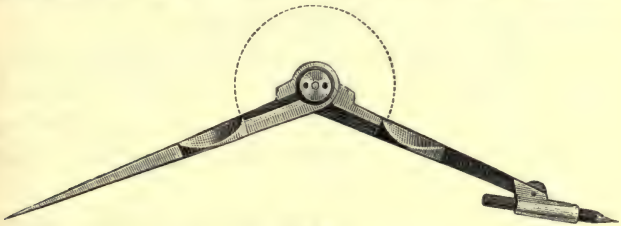


FIG. 28.

bent back, as you might bend back your dividers (Fig. 28).

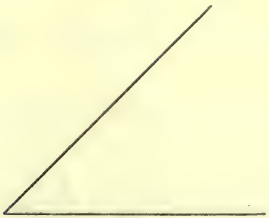


FIG. 29.

The angle in Fig. 29 is half a right angle. What is the reflex angle?

The reflex angle in Fig. 30 is $2\frac{1}{2}$ right angles. What is the ordinary angle?

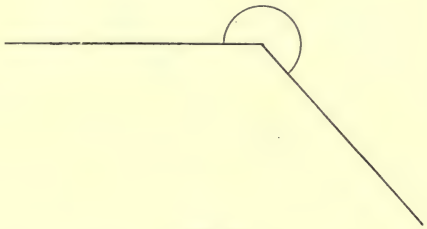


FIG. 30.

5. Face to the north. Then face to the south. What angle have you turned through?

6. At the word of command "About turn," what angle must be turned through? The old words used to be "Right about turn." Draw diagrams showing the angles meant by "Right about turn" and "Left about turn."

7. What angle is meant by the words "Right turn"? How many times must the order "Right turn" be repeated to bring you into the same position as a single order "Left turn"?

8. What is the sum of the two angles A and B? (Fig. 31.)

9. Mention any lines or directions on this page or in the room which are at right angles to each other.

10. Describe a large circle on paper, cut it out carefully, and fold it in two along a diameter. Then fold it again in two. What are the angles between the folds?

11. How many right angles does the long hand or a watch turn through in an hour? In 25 minutes?

12. What fraction of a right angle does the short hand turn through in an hour? In 7 hours?

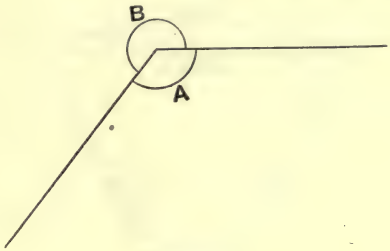


FIG. 31.

13. A right angle is divided into 90 equal angles called **degrees**. A right angle = 90° (90 degrees). In Fig. 32 a **semicircle** is shown with the 2 right angles

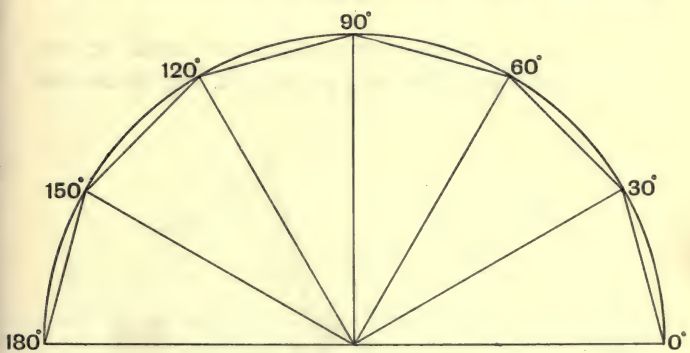


FIG. 32.

at the centre divided into 6 equal parts, each equal to 30°. With your dividers try if the chords are all equal. The meanings of the words **chord** and **arc** are explained in Fig. 33 (*arcus* = a bow, *chorda* = a string).

Are the arcs equal in Fig. 32? How would you measure their lengths?

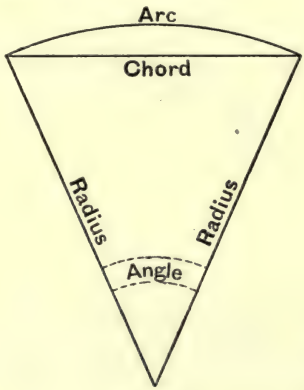


FIG. 33.

14. To test whether two angles are equal or not, you can make use of tracing paper. In Fig. 34 are two

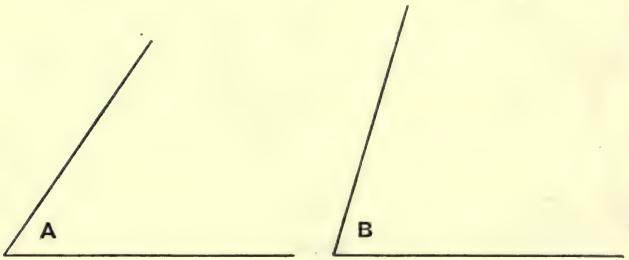


FIG. 34.

angles *A* and *B*. Make a careful tracing of the angle *A*, and place it over the angle *B*. Which is the larger angle?

15. In Fig. 35 the angles C and D are said to be equal. Test the truth of this statement with tracing paper.

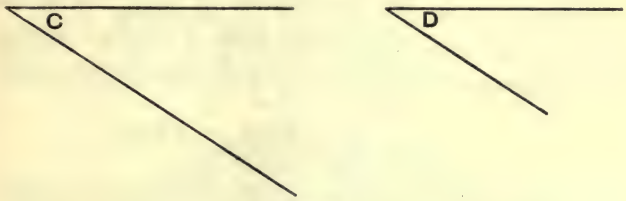


FIG. 35.

16. Sometimes there are several angles with the same angular point, as at A in Fig. 36. When we

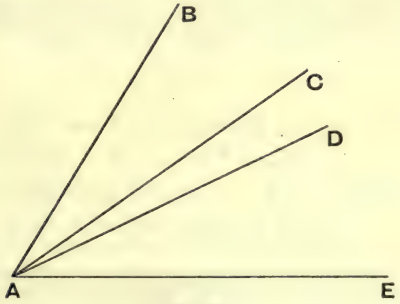


FIG. 36.

speak of the angle A, then, it is doubtful which angle is meant. Therefore three letters are used in naming an angle, the letter at the angular point coming in the middle. Thus this angle (Fig. 37) may be called XYZ or ZYX. There are six different angles in Fig. 36; name each in two ways.

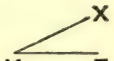


FIG. 37.

FIG. 38.

17. Supposing the Bath road and the York road were straight, the angle between them could be named by the letters of any towns on either road with *L*(*ondon*) in the middle. Thus in Fig. 38 the angle may be called

BLE (or *ELB*),
or *MLY*,
or *BLY*,
or *MLE*.

Name the angles numbered 1, 2, 3, 4, 5 in Fig. 39 in all possible ways.

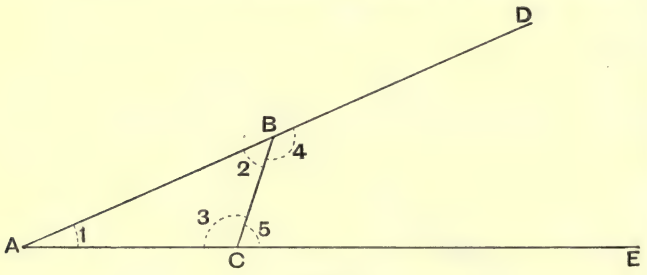


FIG. 39.

18. *AD* is a straight line, *BC* any other straight

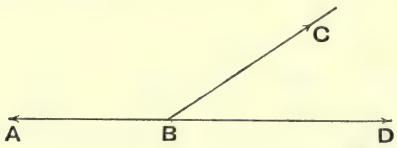


FIG. 40.

line. Set your dividers with one point at *B*, and turn

them first from BD to BC, and then from BC to BA. How many right angles has the moving point turned through? How many degrees?

19. A **protractor** is an instrument for measuring angles. It may be made semicircular or oblong. Both shapes are shown in Fig. 41. The figure also shows

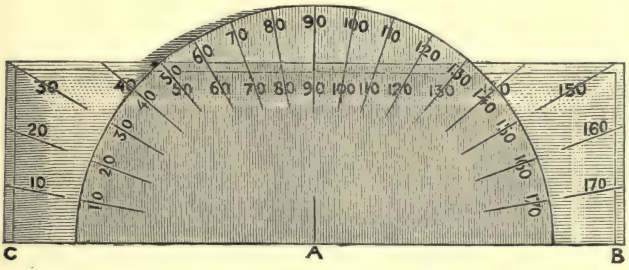


FIG. 41.

how one shape can be obtained from the other. Which shape came first? Give your reasons.

20. In measuring an angle, you should place the protractor with A at the angular point and AB or AC

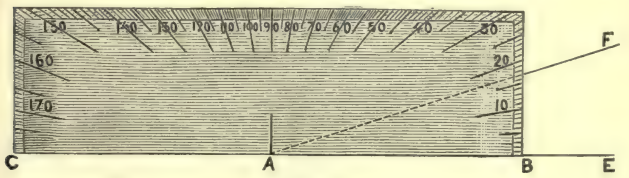


FIG. 42.

along one **arm** of the angle, and count the number of degrees from that arm to the other arm. Thus, in Fig. 42, the angle EAF is 17°.

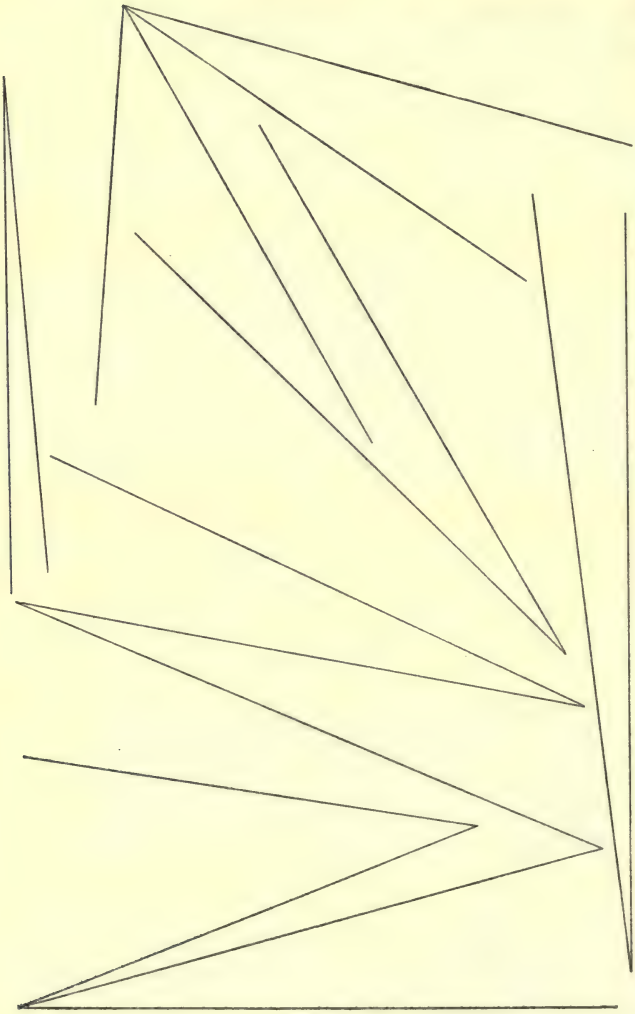


FIG. 48.

21. Measure the angles in Fig. 43.

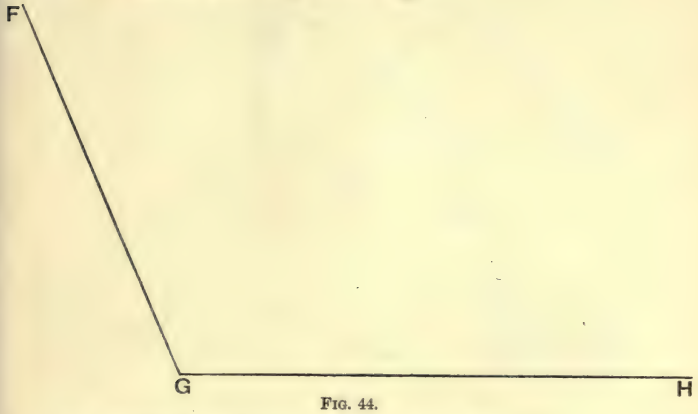


FIG. 44.

22. Measure the angle *FGH* (Fig. 44).

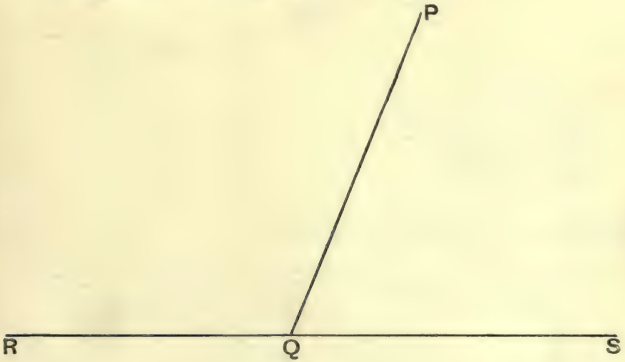


FIG. 45.

23. Measure each of the angles *PQR*, *PQS* (Fig. 45).

24. Why has each division on the protractor two numbers against it? How can you tell which to use? What is the sum of each pair of numbers?

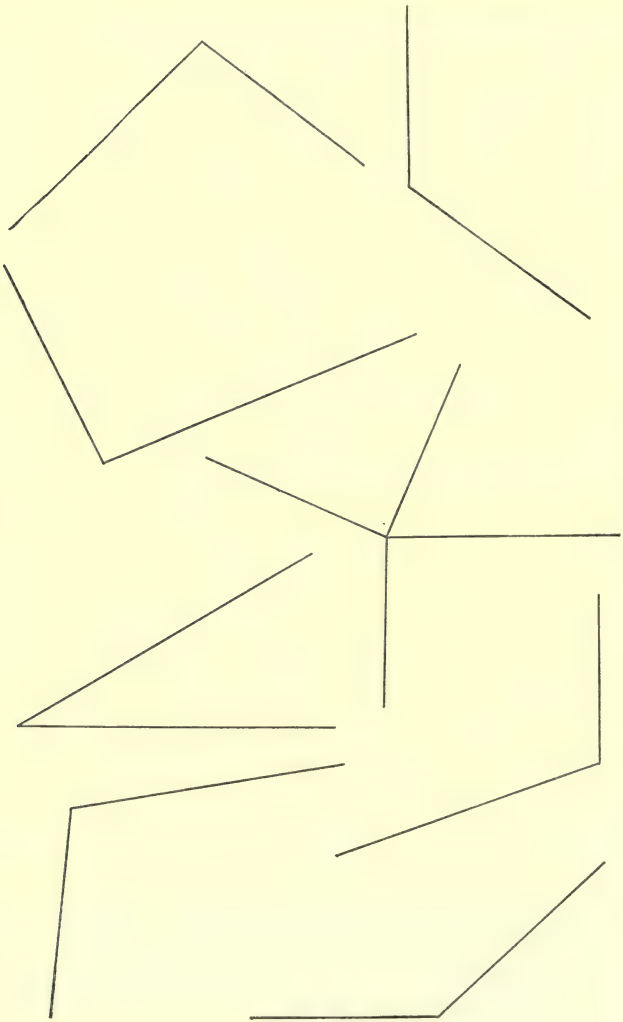


FIG. 46.

25. Measure the angles in Fig. 46.

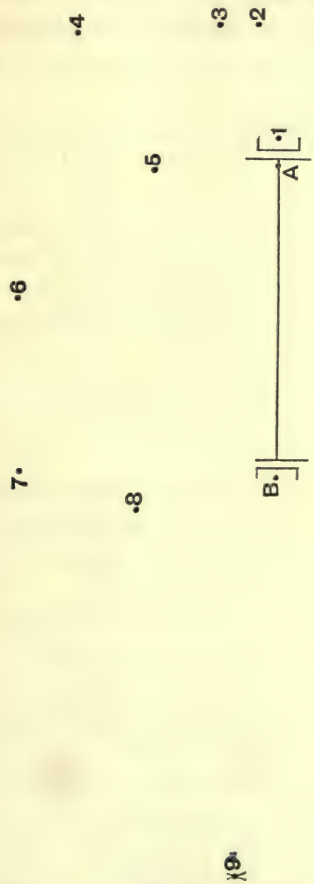


FIG. 47.

26. Fig. 47 shows a plan of a game of cricket. Measure the angles through which the batsman must

turn the ball in sending it to each of the fieldsmen, taking them in numerical order.

27. Your protractor can be used for constructing angles. Draw a line AB 3 in. long, and at each end of it make an angle of 73°.

28. Make a line AB 2 in. long. At A make an angle BAC 39°. At B make an angle ABC 120°. With your rule produce (*i.e.* lengthen) the two lines until they meet in C. Measure the angle at C.

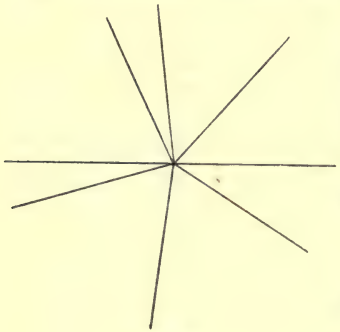


FIG. 48.

29. Measure the angles in Fig. 48, and find their sum.

30. An angle greater than a right angle is called an **obtuse** angle. One less than a right angle is called an **acute** angle.

AB, CD are two straight lines intersecting at E. Name the angles at E, saying whether each is acute or obtuse.

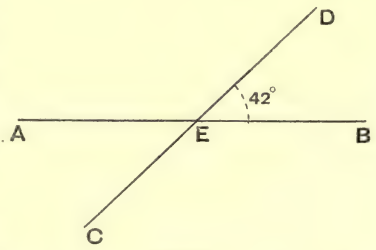


FIG. 49.

31. In Fig. 49 the angle DEB is 42°. Without measuring the other angles, write down the number of degrees in each.

32. The four angles in Fig. 50 are all equal. What is the size of each of them? (Protractor not to be used.)

FIG. 50.

33. Measure the angle which the door **subtends** at your eye, *i.e.* the angle between the directions in which you see the top and the bottom of the door. You can do this roughly by setting your dividers (or compasses) so that when your eye is close to the joint, the legs of the dividers appear to point straight to the top and bottom of the door. Then mark on paper the angle between the legs, and measure it with your protractor.

34. Find at what distance from your eye you must hold one edge of your set square so as just to cover the height of the door. Then by drawing a diagram showing your eye and the edge of the set square with measurements in the right proportion, you can measure the angle as before.

35. The wind shifts from north to east. What angle do the weathercocks turn through? The wind then shifts from east to south-west. What is this angle? Through what angle has the weathercock turned since the wind was north? If the wind had shifted from north to west and then to south-west, what would the complete angle have been?

36. In Fig. 51 are shown the "points of the compass." How many are there?

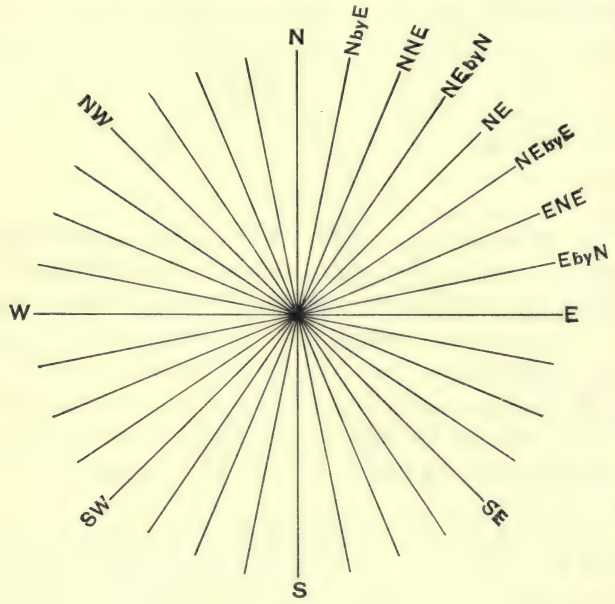


FIG. 51.

If the wind shifts through two points, how many degrees is that?

The names of the points from *N.* to *E.* are given. Give the names from *E.* to *S.*

37. A ship sails 4 miles due north, and then shifts her course four points to the westward and sails another 4 miles. Draw a chart of her course (1 inch to the mile), using your protractor to make the angle.

ADDITIONAL EXERCISES.

V. a. CIRCLES.

1. *A* and *B* are two forts 3 miles apart, and contain guns whose range is 2 miles. Draw a figure to show the space covered by the guns of the two forts.

2. Remembering the meanings of arc and chord, find the chord of 45°, when the radius is 2 inches. (K.)

3. A man walks 3 miles, and then walks in a circle through whose centre he had passed and whose radius is one mile, till he has made the chord of the circle equal to its radius. Find how far he is then from home. (K.)

4. A fly is 1 inch from the centre of a given circle 3 inches in radius; another fly is stationed half-way between the first fly and centre of the circle; let the first fly make for any point you like in the circumference, and find how far he will be from the second fly when he has got half-way. (K.)

5. Describe two equal circles radius $1\frac{1}{2}$ inches, so that the circumference of each may pass through the centre of the other. (K.)

6. Describe three unequal circles so that the centre of each may lie on the circumference of one of the others. (K.)

7. A see-saw, 10 feet long, rests with its middle point on a pivot 4 feet from the ground, and starts from a horizontal position, through what angle will it turn before one end touches the ground; and how far from the ground will the other end be? If both ends touch the ground alternately, what is the distance between the marks?

V. b. ANGLES.

1. A man walks two miles, then turns to his right hand through a right angle and walks for three miles, then turns to his left through half a right angle and walks a mile. Draw a plan, assuming only an inch to represent a mile, and find how far he then is from his starting-point.

Caution:—When it is said that a man turns through 45° to his left it means that he turns through an angle of 45° from *the direction in which he would have gone, if he had gone straight on.* You should therefore produce your line beyond the point where he stops, and the new direction will make an angle of 45° with this line. (K.)

2. A path goes straight for 3 yards, then goes at right angles for 8, then goes straight to the middle point between where it first started and where it turned off. Find how long it is. (K.)

3. A man walks two miles, and then turns through an angle of 60°. [*Caution:*—Here again the angle of 60° is to be laid off from the direction in which he would have gone if he had not turned off.] He then walks 2 miles, turns the same way through the same angle and again walks 2 miles. Find how far he is from where he started. (K.)

4. A man walks 3 miles, turns through 45° to his left, goes half a mile, turns through 90° to his left, and goes 1 mile further. Show that he is about $2\frac{2}{3}$ miles from home. (K.)

5. Three men start from the same point, going N., S., E. at the rates of 3, 4, 5 miles an hour respectively; draw two lines at right angles so as to represent the N., S. and E. directions Find by measurement how far they are from one another at the end of two hours. Scale $\frac{1}{4}$ in. = 1 mile. (K.)

6. The height of a wall is 20 feet; required the length of a ladder which will reach to the top, when the foot of the ladder is 15 feet from the wall. Draw a figure and measure the ladder.

7. If the ladder in question 6 were 20 feet, and its foot 12 feet from the wall, how high would the wall be? (K.)

8. Describe a **square** (*i.e.* a four-sided figure with all its sides equal and all its angles right angles), each side to be 2 inches.

9. *A* and *B* are two forts with a river between them; a man goes from *A* to a bridge, *C*, 10 miles off: then goes to *B* from the bridge, 12 miles, by a road making an angle of 30° with the road from the bridge to *A*. Draw a plan, and find how far *A* is from *B*. (K.)

10. The angle of elevation of an object above an observer means the angle between a horizontal or level line in a direction towards the object and a line elevated upwards and passing through the object.

Similarly **the angle of depression** of an object below the observer means the angle between a horizontal line towards the object and a line depressed downwards and passing through the object.

Find the height of a tower whose angle of elevation 200 yards off is 15°. (K.)

11. A tower stands on a rock; a man from the sea at 100 yards from the foot of the rock finds the angle of elevation of the *foot* of the tower 15°; he then rows 100 yards further off and finds the elevation of the *top* of the tower 15°. Find the heights of the rock and of the tower. (K.)

12. At the top of a mountain, the angle of depression of the peak of a neighbouring mountain is 5°: if the difference of the heights of the two peaks be known to be 500 feet, find the distance from the centre of the base of one mountain to that of the other. Scale $\frac{1}{2}$ in. = 500 feet. (K.)

13. The distance between wickets is 22 yards; Long-on and Long-off are on opposite side of the line between wickets; the line joining either of them to the bowler's wicket is 22 yards long, and subtends an angle of $22\frac{1}{2}°$ at the batsman's wicket; how far are they from one another? Coverpoint is the third point of a triangle whose base is the line of wickets, and whose base angles are 75° at the batsman's and 60° at the bowler's wicket. Find how far he is from Point, who is on the same side of line of wickets, distant 8 yards from the batsman's wicket, in a direction perpendicular to the line between the wickets. (K.)

14. A man measures a four-sided field; he finds that the distances across it from the opposite corners are 100 and 120 yards; that these two distances bisect one another in the middle, and are inclined at an angle of 30° to one another. Find the length of the shorter side. (K.)

15. Two roads branch off at A, making an angle of 45° with each other. Find the distance of two places, one being 7 miles

from A along one road, and the other 8 miles from A along the other. (K.)

16. A tower subtends angles of 46° and 51° at points in a straight line with the foot of the tower and distant 10 yards apart. Find the distances of the two points from the tower.

17. A is a point due west of a train on a straight line of railway which runs north-west. After the train has travelled 4 miles A is S.S.W. of the train. Find the distance of A from the train at the first observation.

18. A tower 40 feet high stands on a mound. At a point on level ground the angles of elevation of the top and the bottom of the tower are 41° and 29°. Find the height of the mound.

If on the top of the tower there is a flagstaff subtending the same angle as the tower, what is its height?

19. AB is a line 1000 yards long, B being due east of A. At A an object C is seen to bear 30° north of east, and at B it bears 45° north of east. Find the length of AC and BC.

20. Draw a picture of the Compass, marking the N., W., S.W., and S.W. by W. points. What fraction of a right angle is there between the S.W. by W. and N.W. by N. points? Two men start together, going one N.W. at 40 miles an hour, the other S.W. by S. at 30 miles an hour; how far apart will they be after ten hours? (K.)

21. A flagstaff stands on level ground; at two points in the same straight line through its foot and 48 feet apart the angles of elevation of its top are 30° and 60°; find the height of the flagstaff.

22. An observer on the top of a tower 100 ft. high observes the angles of depression of the opposite banks of a river, which runs perpendicular to the direction in which he looks, are 57° and 43°. Find the breadth of the river.

23. Three ships A, B, C, start together from a port, A going south, B south-west, and C west; and their rates are such that they keep in a straight line with one another. If A's rate is 10, and C's 6 miles an hour, find B's rate. (K.)

24. From the top of a vertical cliff, the angle of depression of

a point on the shore 150 feet from the base of the cliff, is observed to be 30° : find the height of the cliff.

25. From two points distant 40 feet apart and in the same straight line as the base of a tree, the angles of elevation of the top of the tree were found to be 50° and 33°. Find the height of the tree and the distance of the two points from the base.

26. From station B at the base of a mountain its summit A is seen at an elevation of 40°. After walking one mile towards the summit up a plane making an angle of 15° with the horizon to another station C, the angle BCA is found to be 130°. Find the distance CA.

27. Find the distance between two inaccessible objects, which can be seen together from one point, only from the following notes :

Let A, B be the objects, C the point from which both can be seen. The angle DCE is 135°. I measure CD and CE, each 100 yards ; and observe the angles CDA and ACD, and find them 30° and 80°. I measure the angles BCE and CEB, each $67\frac{1}{2}°$. (K.)

CHAPTER VI.

CONSTRUCTION OF PARALLELS AND PERPENDICULARS BY SET SQUARES.

1. In Fig. 52 which is the larger angle, *EBC* or *DAC*? Guess the number of degrees in each. Then measure the angles with your protractor. If the lines

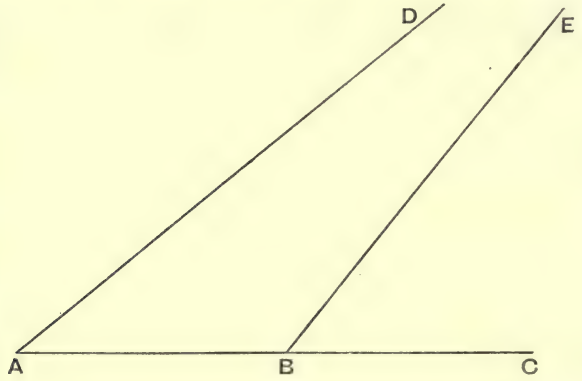


FIG. 52.

AD, *BE* were produced, would they ever meet? If so, would they meet above or below the line *AC*?

2. Answer the same questions as in Exercise 1 for Fig. 53.

3. In Fig. 54, the straight lines AD, BE will not meet either way. The angle EBC is 50°. Write

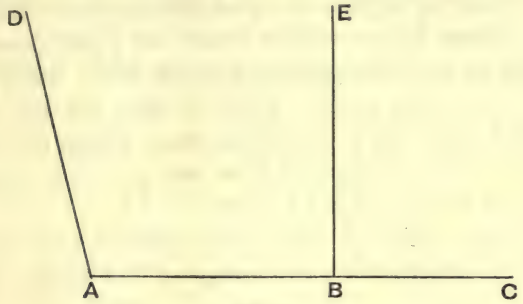


FIG. 53.

down, without measuring them, the size of the angles EBA, DAB. The angles EBC, DAB may be called **corresponding angles**.

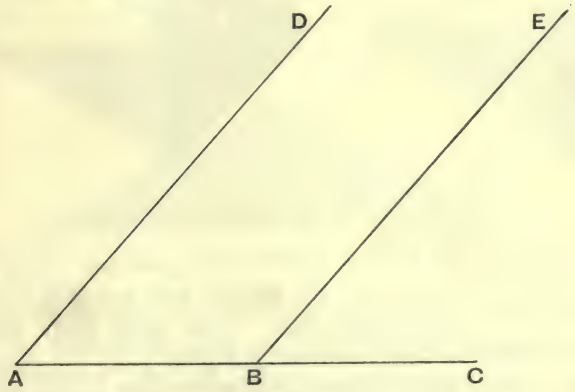


FIG. 54.

4. Two lines like DA and BE which never meet if produced either way are called parallel lines

($\pi\alpha\rho'$ $\dot{\alpha}\lambda\lambda\eta\lambda o\hat{\iota}\nu$ = alongside each other). Name any lines in the room which are parallel to one another. (When corresponding angles are equal, the lines are parallel.)

5. Draw two straight lines on your paper as nearly as you can parallel to each other, using only your ruler and pencil. Even if they are not quite parallel, they will probably not meet within the limits of your sheet of paper. How can you test whether they are parallel or not? (See Ex. 3.)

6. Take one of your set squares and rule a straight line with one of its edges AB (Fig. 55). Now

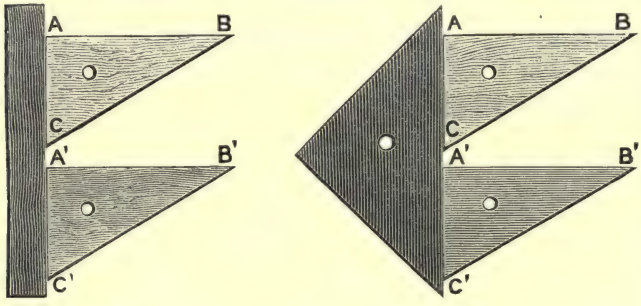


FIG. 55.

holding the set square in the same position, bring up your scale, or another set square, and lay it against one of the other edges AC. Now keep your scale firmly fixed in this position, and slide your set square down into another position $A'B'C'$, taking care that the edge AC is up against the scale. Now rule the line $A'B'$. This line should be accurately parallel to the line AB. Why?

7. Practise ruling parallel lines until you can draw them quite easily, and then rule five lines parallel to one another.

8. How did you make a right angle with a piece of paper? How could you tell by folding if the line

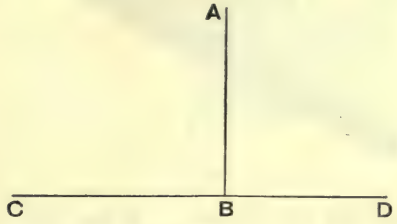


FIG. 56.

AB is at right angles to the line CD? Copy the figure on tracing paper, and make the experiment.

9. To find out if your set square is accurately right angled, rule a line AB (Fig. 57) and take a

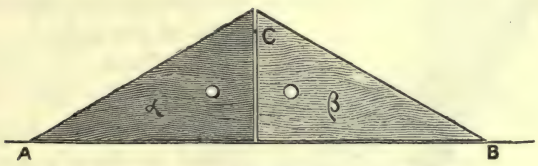


FIG. 57.

point C outside the line. Then lay your set square in the position α with one edge of the supposed right angle along AB, and with the other edge rule a line through C. Now turn the set square over to the position β and again rule a line through C. This ought to be the same as the first.

10. Practise ruling parallels as in Fig. 58, keeping the longest edge of the set square against the ruler. At the same time rule lines with the edge AC in each

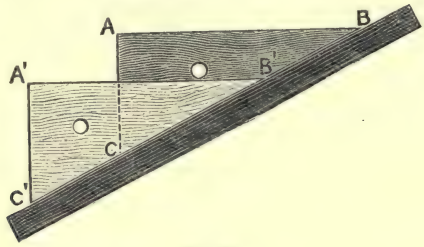


FIG. 58.

position of the set square. What are the angles between your two sets of lines?

(If your ruler has a bevelled edge, you will find that your set square is liable to slip over or under it. In this case you had better use another set square instead of the ruler.)

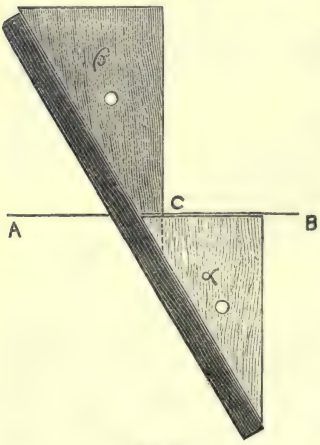


FIG. 59.

11. Draw a straight line AB (Fig. 59). Take a point C in it. It is required to draw a straight line through C perpendicular (*i.e.* at right angles) to AB. Put the set square in position α. Then bring up the ruler as in Fig. 59 and slide the set square up it into the position β. Then rule your line through C.

Why should not the set square be placed at once with the right angle at C as shown in Fig. 60 (i)? Because the perpendicular so drawn is inaccurate

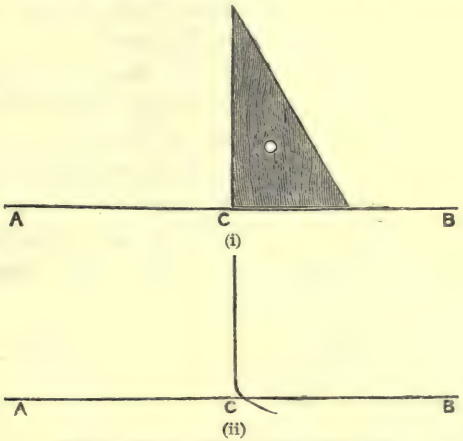


FIG. 60. Wrong method of using set square.

near C, owing partly to the slipping of the pencil and partly to the tendency of the corners of set squares to get worn down and rounded.

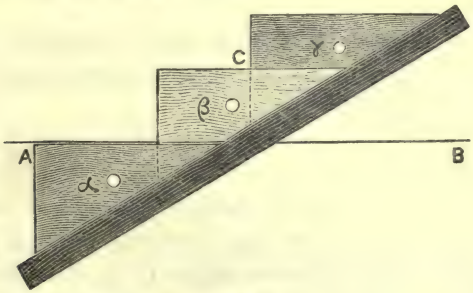


FIG. 61.

12. Draw a straight line AB (Fig. 61), and take a

point C outside it. Through C draw straight lines parallel and perpendicular to AB.

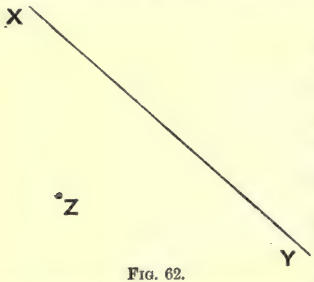


FIG. 62.

The positions of the set square are marked α, β, γ in Figure 61.

13. Rule a straight line obliquely across your page, as in Fig. 62, and take a point Z outside it. Through Z draw by eye, as nearly as you can, straight lines parallel and perpendicular to XY. Then test their accuracy with the set square.

14. From a point A outside BC draw $AD \perp$ (perpendicular) to BC and other lines AE, AF, AG. Measure their lengths. Which is the shortest? Which is the longest?

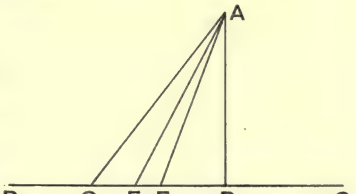


FIG. 63.

15. **The distance of a point from a straight line** is the length of the perpendicular drawn from that point to the straight line.

Measure the distance of A from BC in every position in Fig. 64.

16. Rule two parallel straight lines, and various other straight lines reaching from one parallel to the other and making different angles (Fig. 65). Measure these lines. Which is the shortest, and which the longest?

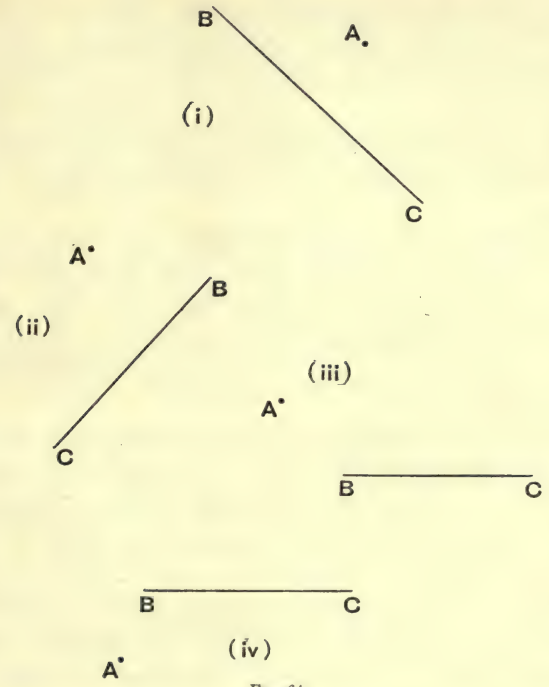


FIG. 64.

How would you draw the shortest possible line from

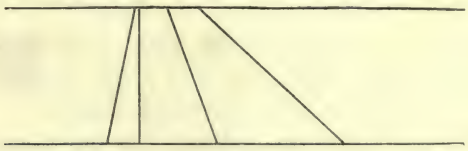


FIG. 65.

one parallel to the other? Find the **distance between** the two straight lines in Fig. 65.

17. When two parallels are said to be 1 inch apart, the perpendicular distance between them is 1 inch. Rule five parallels at distances of 1 inch, using the inch ruler and set square as in Fig. 55 (i).

18. Rule a set of parallels 1 cm. apart, and another set crossing the first at right angles and also 1 cm. apart.

19. Describe a circle of radius two inches, and rule a diameter. Then with your set square rule a second diameter at right angles to the first. Measure the distances from the ends of one diameter to the ends of the other (to the nearest hundredth of an inch).

20. Rule a straight line AB 4 inches long, and at A make the angle $BAC = 67°$. Produce AC so as to make it 3 inches long. Through B rule a straight line parallel to AC, and through the end of AC rule a straight line parallel to AB. Let these two lines intersect in D. The figure $ABCD$ is **quadrilateral** (*i.e.* has four sides), and its opposite sides are parallel. A figure of this kind is called a **parallelogram.**

Measure the sides CD, BD.

Also measure the angles of the figure.

21. Make a parallelogram with two of its sides 7 cm. and 5 cm. long, and the angle between them 105°. Measure the other sides and angles.

22. The straight line joining the opposite corners of a parallelogram is called a **diagonal.** Draw the diagonals of your parallelogram in Ex. 21, and measure them. Are they equal?

23. A parallelogram with one angle a right angle is called a **rectangle**. Use your set squares to make a rectangle of any size you like. Measure its diagonals. Are they equal?

24. A rectangle with all its sides equal is called a **square**. Make a square of side 2 inches. Measure its diagonals.

25. Make a rectangle with sides 4 inches and 3 inches long, and using your set square as in Fig. 55 (i), and sliding it down 1 inch at a time, divide the rectangle into square inches. How many square inches does it contain?

26. Make a rectangle with its sides 7 cm. and 5 cm. long, and, by means of the set square and the centimetre scale, divide it up into square centimetres.

27. Give instances of rectangles that you can see in the room.

ADDITIONAL EXERCISES.

VI. a.

1. Draw AB 3·6 in. long, AC at right angles 3 in. long. Join BC and from A draw AD perpendicular to BC. Measure AD, and the angle BAD.

2. Draw AB 3·8 in. long, make the angle BAC 65°, draw AC 4·5 in. long. From B and C draw BD, CD at right angles to AB and AC respectively to meet at D. Join AD and measure its length.

3. Draw a plan of two goal posts 18 feet high, and 24 feet apart, with a cross bar 10 feet from the ground. (K.)

4. Draw a ground plan, finding a spot whose distance from the goal line is 12 yards, and whose distance from the further

goal post is 20 yards, and measure its distance from the nearer goal post. (K.)

5. Find at what angle of elevation a ball must be kicked from the spot above mentioned so as just to hit the nearest point of the cross bar. (K.)

6. Take any triangle inscribed in a circle, and from any point in the circumference of the circle drop perpendiculars on the sides of the triangle, or its sides produced; the feet of these three perpendiculars (that is, the points at which they meet the sides produced) will be in the same line. (K.)

CHAPTER VII.

MEASUREMENT OF AREA.

1. In a nursery garden there are some young trees, each of which requires a square space of 1 yard each way. There are two plots of ground, one of them 4 yards each way, and the other 5 yards long and 3 yards broad. Which can grow most trees, and how many can be grown in each?

2. Make plans of the two plots of ground, scale 1 inch to the yard, and show them divided up for planting. Just as we measure a length by dividing it up into a number of equal lengths, such as inches or centimetres, so we measure an area, that is, a space which has breadth as well as length, by dividing it up into equal areas. The most convenient way is to divide into squares (Fig. 66.)

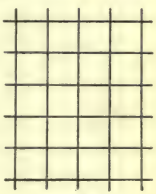


FIG. 66.

3. In Fig. 67 there is an area of irregular shape drawn on squared paper. The large squares are square inches.

How many of the small squares make one square inch?

What is each of the small squares? Write it as a decimal of a square inch.

4. Find the area of the figure as nearly as you can by counting the small squares and reducing to square inches. Use any method of saving time in counting, such as marking off rectangles in pencil.

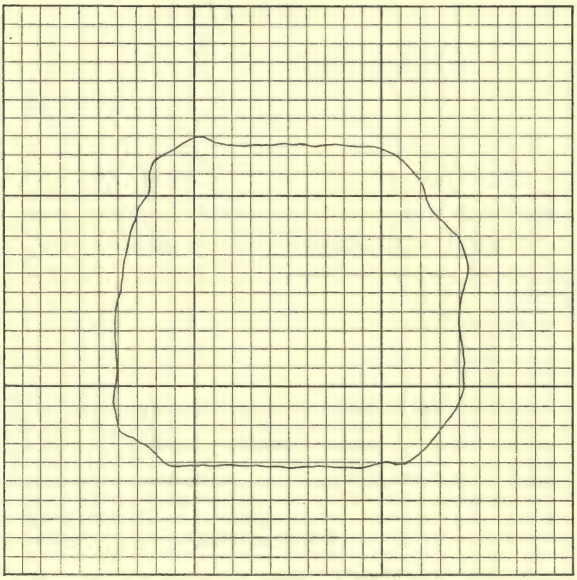


FIG. 67.

Any fraction of a small square that looks less than a half need not be counted, any that looks greater than a half may be counted as a whole square.

5. Describe on squared paper a circle of radius 2 in. Find its area as in Exercise 4.

6. Draw a square 3 cm. each way, and divide it

up into square cm. Your figure should tell you the number of square feet in a square yard.

7. Draw a square of 12 cm. (or half centimetres if you prefer it), and divide it so as to illustrate the number of square inches in a square foot.

8. On your squared paper suppose each tenth of an inch to represent 1 foot. Mark off on the paper a picture of a rectangle 24 feet long and 15 feet broad (see VI. 23). How many square feet does it contain? Is it necessary to count the squares one by one?

9. If you are measuring length in inches, 1 inch is called the unit of length. Similarly, in measuring a surface, 1 square inch would be the unit of area. Supposing we measured length in tenths of inches, what is the unit of area?—one-tenth of a square inch?

10. Taking $\frac{1}{10}$ inch as unit of length, mark off on the squared paper a rectangle 29 by 17. What is its area? Now measure the length and breadth in inches (and decimals of an inch). What is the area in square inches?

11. Without drawing the rectangle, calculate the area of one the length of which is 3·7 inches and breadth 2·3 inches.

12. You have found the number of centimetres in an inch (II. 13); hence calculate the number of square centimetres in a square inch.

13. Measure the length and breadth of the rectangle on the next page (Fig. 68) as accurately as possible in inches, using your dividers, and hence calculate its area in square inches.

14. Suppose you measure the length and breadth of a rectangle and find it 3·47 in. by 2·93 in. Then the area is $3{\cdot}47 \times 2{\cdot}93$ square inches, and this gives 10·1671 square inches. But you may have been wrong in each of your measurements as much as $\frac{1}{100}$th of an inch, *i.e.* you are not absolutely certain that the lengths were not 3·48 in. and 2·94 in. Now $3{\cdot}48 \times 2{\cdot}94 = 10{\cdot}2312$. It is plain then that you cannot be certain of the area

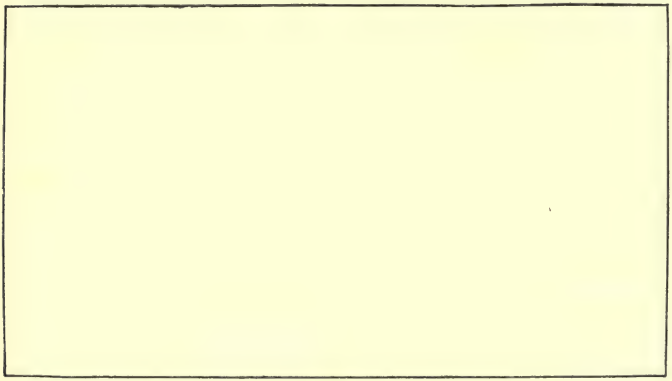


Fig. 68.

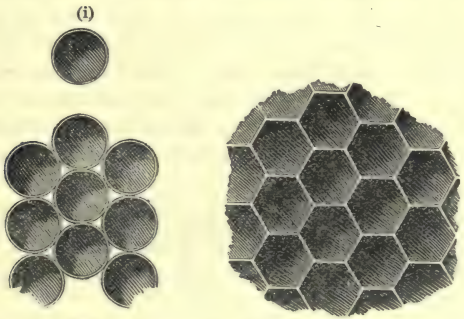
at any rate beyond the first place of decimals, and you should therefore write the area as 10·2 square inches, ·2 being the nearest tenth of a square inch. Measure the rectangle in Fig. 68, using the edge of your rule instead of the dividers. To how many places do your results agree with those obtained in Ex. 13.

15. Measure the rectangle in Fig. 68 in centimetres, and calculate its area in square centimetres. To how many places may your result be considered accurate?

16. From your results in Exercises 13 and 15, find the number of square centimetres in a square inch, and tabulate your results as in II. 15.

17. Find the area of the cover of this book in square inches and in square centimetres, and hence find the number of square centimetres in a square inch. Does your result agree with previous results?

18. A solitary bee's cell is round, but in a hive the cell-floor is divided into **hexagons** (Fig. 69). Can you



(i) Solitary bee's cell. In a hive the cells are not as in (ii), but as in (iii).
FIG. 69.

give any reason for their using hexagons and not circles? Can you give a reason for using squares and not hexagons in reckoning area?

ADDITIONAL EXERCISES.

VII. a.

1. Draw a line one inch long and construct a square upon it. Draw also lines two and three inches long and construct squares upon them. Divide each of these figures into square inches, and write down the number of square inches each contains.

2. How many square inches are there in a square drawn upon a line four inches long? How many in squares having sides 6 inches and one foot long?

3. Draw an **oblong** (or rectangle) 4 inches wide and 3 inches high. Divide the figure into square inches and count the number there are in it. Repeat the exercise with an oblong 5 cm. wide and 4 cm. high.

4. Mark off lengths an inch apart from one corner along the bottom and top edges of your exercise book or a sheet of paper. Draw straight lines up the page from each of these marks to the one above it. From the same corner mark off lengths of inches on the outside and inside edges of your book and connect corresponding points. You will thus divide the paper into square inches. Count the number of complete square inches you have drawn. Then notice the length in inches from the corner of the book to the last line along the bottom and outside edges. Multiply these two lengths together. Put down your results thus:

Number of square inches in page - - - -
Breadth from corner to last inch mark on bottom -
Height from corner to last inch mark on outside edge - - - - - - - - -
Breadth × height

5. Construct a figure to prove that a square having each side half an inch long has an area of one quarter of a square inch.

6. Prove, by constructing a figure and dividing it into squares, that the area of an oblong is equal to the product of two adjacent sides.

7. Find by construction and calculation, the area in square inches of an oblong $2\frac{1}{2}$ inches wide and $1\frac{1}{2}$ inches high.

8. Draw a line 1 dm. or 10 cm. long and construct a square upon it. Divide the figure into square centimetres. Count the number of square centimetres in one square decimetre.

9. Draw a square inch and divide it up into square centimetres. How many square centimetres are equal in area to 1 sq. inch?

10. The length of an inch is equal to 2·54 cm. Find by calculation the number of square centimetres equal to 1 sq. in.

11. Draw two oblongs of any convenient size. Measure the lengths of adjacent sides in inches and from the measurements calculate the area. Repeat the measurement in centimetres. Use your results to find the number of square centimetres in one square inch, thus :

Area of a given rectangle in square inches.	Area of same rectangle in square centimetres.	Square centimetres / Square inches

12. How many square feet are there in the floor of a room 19 feet long and 17 feet broad ?

13. Measure the length and breadth of your table and make a plan of it, letting half an inch on your paper represent one foot length of the table. How many times broader is the table than your plan of it ? If you cut out pieces of paper the size of your plan, how many would be required to cover the top of the table ?

14. Draw a square centimetre, and a square inch. How many times greater is one square than the other ?

15. Suppose each of the little squares on your squared paper to represent 1 acre. Draw lines so as to enclose 640 acres, that is 1 square mile.

16. If 8000 metres are equal to 5 miles, how many square metres are there in an acre ?

17. Find the length of carpet 2 feet wide required for carpeting a floor of a room, 24 feet by 20 feet, allowing a margin of 3 feet wide for staining.

18. Find the cost of a square field 550 yards long at £52. 6s. an acre ; and the area of a path 3 yards wide running outside it.

19. How much carpet 27 inches wide would be required for a room 23 feet by 32 feet, a margin of 4 feet being left uncovered?

20. Find the number of pieces of paper 12 yards long and 2 feet wide required for papering the walls of a room 19 feet 6 inches long, 17 feet 4 inches wide, and 10 feet 6 inches high, allowing $173\frac{1}{2}$ square feet for doors and windows.

21. A strip of building land on the side of a straight road has a depth of 180 feet. It is sold at the rate of £5. 5s. per foot of road-frontage. What price is this per acre?

22. Find the length of the side of a square field containing $2\frac{1}{2}$ acres.

23. How long will it take to walk round a square field of 13 acres 81 square yards at the rate of $3\frac{3}{4}$ miles an hour.

24. A running track is laid round a square 10-acre field. How many laps would go to the mile?

25. Two adjacent sides of a rectangle are measured and are found to contain 1·534 metres and 0·662 metre respectively; find its area.

If in each measurement there may be an error of 2 millimetres, what is the greatest possible error in the area of the rectangle as calculated from the above measurements?

CHAPTER VIII.

VOLUME.

1. Fig. 70 shows a drawing of a rectangular block of wood. Its dimensions are written on it. Some of the faces of the block are visible, but of course there are others that are not shown in the picture. How

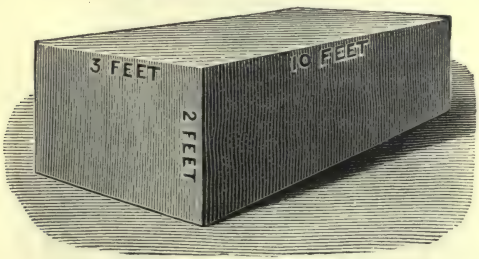


FIG. 70.

many faces has the block altogether, and what is the area of each of them? Be careful to state what you are measuring in, *i.e.* what your unit of measurement is.

2. Can you answer the question: "How much timber does the block contain?" If so, what is your unit of measurement?

3. Supposing Fig. 70 to represent a tank full of water instead of a block of timber, the inside length,

breadth, and thickness of the tank being the same as in Fig. 70. Now suppose the water to be let out into another tank of quite a different shape. The length, breadth, and depth of the water would be altered. What would remain the same?

4. Examine your wooden cubes. A **cube** ought to have its length, breadth, and thickness the same; and a cube 1 foot each way is called a cubic foot. Measure one of your cubes to see if it is perfect, and state its size. How many faces, or surfaces, has it?

How many edges?

How many corners?

How many edges meet at each corner?

How many surfaces meet in each edge?

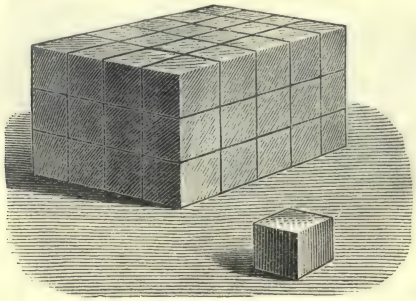


FIG. 71.

5. Make a pile of cubes two each way. How many cubes form the pile? How many form the base of the pile?

6. If you had to make larger cubes 3, 4, 5, 6, 7 each way, state in each case how many would be wanted, and how many would form the base.

7. We measure **area** by dividing it up into squares. We measure **volume** or **cubic content** by dividing it up into cubes. Fig. 71 shows a block 5 inches long, 4 inches wide, 3 inches thick, with a shaded picture of a cubic inch at the side. Imagine the block divided into layers.

How many cubes in the top layer? How many layers? How many cubes altogether?

Divide the block into slices parallel to (i) the upright face in shadow, (ii) the other upright face. How many in each slice? How many slices? Total number of cubes?

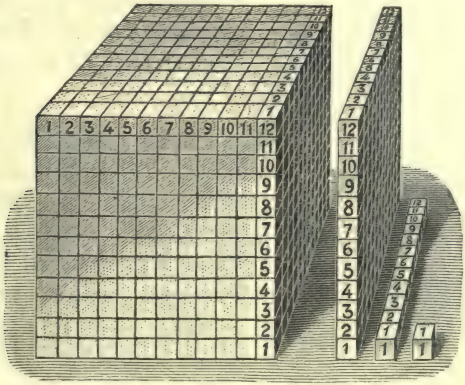


FIG. 72.

8. From Fig. 72 you can find the number of square inches in a square foot (12^2) and the number of cubic inches in a cubic foot (12^3). What is the number of square centimetres in a square decimetre? How many square mm. in a square cm.? in a square

metre? How many cubic centimetres in a cubic decimetre? How many cubic mm. in a cubic metre?

9. A block is 12 ft. long, 10 ft. broad, 5 ft. high. What is its volume?

10. A rectangular trough is 5 yards long, 2 ft. wide, 1 ft. 6 in. deep. Find its cubic content in cubic inches, or in cubic feet.

11. Make a careful paper model of a cubic inch, *i.e.* a piece of paper which will fold up into the shape of a cubic inch. Fig. 73 shows the arrangement, though the lengths are not inches. The shape should be carefully drawn on paper, and then cut out. The dotted lines in Fig. 73 are the lines of fold.

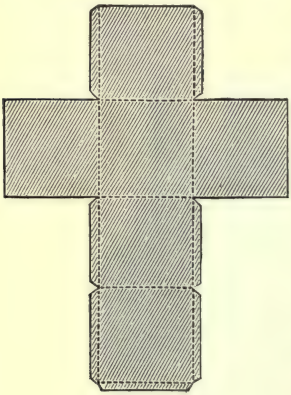


FIG. 73.

12. Measure the block of wood given you in centimetres and decimals of a centimetre, and give the following: length, breadth, thickness, area of each face, volume, and area of the whole surface of the block.

13. Stand the block on its smallest face. Mark out on paper carefully the shape and size of a piece of paper which would just cover the 4 upright faces.

14. The dimensions of a room are 20 ft. by 15 ft. by 12 ft. Taking $\frac{1}{10}$ inch to represent 1 foot, mark out a piece of paper to show the four walls of the room: also another piece of paper for the ceiling.

15. An inch contains 2·54 centimetres. Find the number of cubic centimetres in 1 cubic inch. To how many places of decimals is your result correct?

16. Measure the volume of the block of wood (Exercise 13) in cubic inches. Then find, by dividing your result in Exercise 13 by this result, the number of cubic centimetres in 1 cubic inch. Tabulate your results as in II. 15.

17. The following units of measurement are used in measuring length, area, and volume. State for which of the three each is used: Fathom, acre, pint, bushel, square mile, chain.

18. In what units would you measure the following quantities? In each case state the most convenient unit:

The amount of canvas required to cover a box.

The amount of breathing-space in this room.

The amount of land under pasture in Ireland.

The amount of gravel required to cover a garden path to a depth of 3 inches.

The amount of carpet required for this room.

The amount of carpet 2 feet wide required for this room.

The amount of paper required for the walls.

The amount of space required for a swimming bath.

The amount of water required to fill the bath.

19. What measurements would you take to find the number of bricks used in building a wall?

20. What measurements would you take to find the seating accommodation of a hall?

21. In Fig. 74 is shown a cubic centimetre of lead and a vessel for measuring cubic centimetres of water. How can the accuracy of the graduations of the vessel be tested with the leaden cube?

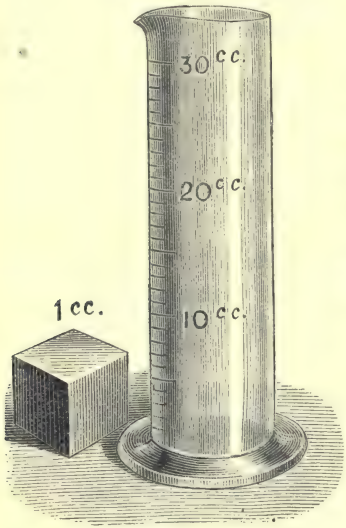


FIG. 74.

22. The volume of a solid can sometimes be found by noticing how much water it displaces. For instance, suppose a piece of lead put into the flask in Fig. 74 sent the water up from 20 c.c. to 27 c.c., what would be the volume of the lead?

23. If you were given a piece of metal too large to put in your measuring flask, but not too large to go into another glass vessel with a mark on it (Fig. 75), how could you measure the volume of the metal?

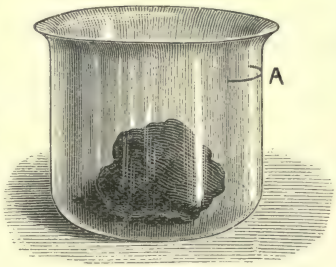


FIG. 75.

ADDITIONAL EXERCISES.

VIII. a.

1. If the measurements of a room are 50 ft. × 40 ft. × 20 ft., how many cubic feet does it hold? How much paper would be wanted to cover the walls?

2. A room measures 40 ft. × 30 ft. × 20 ft. How many cubic feet does it hold? How much paper would be required to cover the walls.

3. A block of wood measures 5 inches by 4 inches by 3 inches. How many cubic inches could be cut out of it? How much paper would be wanted to cover the whole of the outside of it?

4. Describe a method of measuring the number of cubic centimetres contained in a piece of metal, or any solid that sinks and does not dissolve in water.

5. What length must be cut off from a beam 22 inches wide and 20 inches thick, so as to contain a volume $6\frac{7}{8}$ cubic feet?

6. A wall is 15 yards long, 7 feet high, and 13 inches thick. Find how many bricks are required if each occupies 108 cubic inches. (The space occupied by mortar to be neglected.)

7. A square room has a side 14 feet and height 10 feet 6 inches. Find (1) the number of cubic feet of air in the room, (2) the cost of papering the walls at 1s. 6d. a square yard.

8. How many tons of water are there in a canal 8 feet deep, 21 feet wide, and 10 miles long, if one cubic foot of water weighs 1000 oz.?

9. A kilogram is the weight of a cubic decimetre of water, and a decimetre is 3·937 inches. It being given that a cubic foot of water weighs a thousand ounces, express the kilogram, in pounds avoirdupois, correctly to two places of decimals.

10. Find the volume of water in a tank 120 yards long, 84 yards wide, and 20 feet deep.

If 1000 ounces is the weight of one cubic foot of water, how many tons are there in the tank?

11. A tank 48 feet long, $37\frac{1}{2}$ feet wide, $16\frac{1}{2}$ feet deep, is emptied in half-an-hour by a pipe whose sectional area is 44 square inches; how fast does the water flow in the pipe?

12. Find the volume of a cube, each of whose faces has an area of 1 square foot 81 square inches.

13. Find the cost of lining with lead a cistern whose internal dimensions are 7 feet in length, 3 feet in breadth, and 3 feet 6 inches in depth, the cost of sheet lead being 7d. per square foot.

14. Find the weight in tons per square mile of a rainfall of 4 inches, assuming that 1 cubic foot of water weighs 1000 oz.

15. How many cubic inches of wood are employed in making a box with external measurements 2 feet, 1 foot 10 inches, and 1 foot 8 inches, the wood being 1 inch thick?

16. A golf tee box is made of wood 1 inch thick, without a lid, and its external dimensions are: length 16 inches, breadth 14 inches, and height 13 inches. When empty it weighs $\frac{4}{25}$ of the weight when full of sand. Compare the weights of equal volume of sand and wood.

17. A cube of stone whose edge is 3 feet is totally immersed in a cistern $13\frac{1}{3}$ feet long, 9 feet broad, and 5 feet high. Find at what level water within the cistern must originally stand so that when the stone is immersed the water may be up to the brim.

18. Find in cubic metres the content of a block of stone 2 metres long, 2 metres broad, and 1 metre 25 centimetres deep.

19. There are two cubical blocks of stone. The edge of the one is $3\frac{1}{4}$ inches, and the area of the side of the other is $12\frac{1}{4}$ square inches. Find the difference in their two volumes.

CHAPTER IX.

SURFACES.

1. If you examine the surface of still water in a basin or in a pool, you will find that, away from the extreme edge, it is perfectly level in every direction. You can find this out by floating a straight stick on the water, and noticing that the stick lies along the surface of the water, in whatever direction it may be pointing. Fig. 76 shows two positions of the stick,

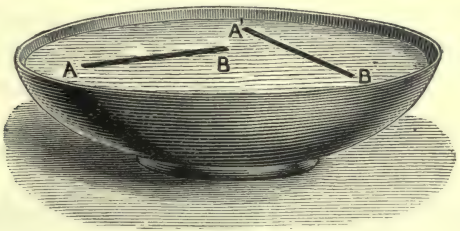


FIG. 76.

AB, *A'B'*. A surface of this kind is called a **plane surface.** (Plane = flat or level.) Sometimes the name plane surface is shortened and the word **plane** alone is used. Now with your ruler, which has already been tested for straightness (III. 4) test the surface of your table or desk to see whether it is plane or not. Describe as clearly as you can how you make the test.

2. Test the covers of this book in the same way.

3. The back of this book is straight, but the surface of the back is not plane. How can you show this?

4. Where two lines cross or **intersect**, we get a **point**. Where two surfaces intersect, we get a **line**. When two plane surfaces intersect, what sort of line is formed by their intersection? You can try this by dipping a flat card into still water at different angles, and noticing the water-line.

5. A round cedar pencil is dipped into water, (i) upright, (ii) **obliquely** (at a slant). Of what shape is the water-line in each case?

6. If you examine a cube, or a rectangular block, you will find (Fig. 71) that it has six plane surfaces, and these intersect in twelve different edges, all of them straight lines. Now examine the solids provided, and state how many different surfaces each has, whether these surfaces are plane or curved, also how many edges, and whether the edges are straight or curved.

(i) A cylinder (*e.g.* a round cedar pencil, unsharpened).

(ii) A cone (*e.g.* the point of a cricket stump).

(iii) A cylinder surmounted by a cone (*e.g.* a neatly sharpened round cedar pencil).

(iv) A sphere (a perfectly round ball).

(v) A hemisphere.

(vi) A lens.

(vii) A hexagonal prism (*e.g.* a hexagonal cedar pencil).

(viii) A pyramid.

7. Place your pencil upright on your paper. Is the pencil at right angles to every direction in the paper? Test this by putting the right angle of your set square against the pencil and turning it round the pencil, and noticing if it fits in between the paper and the pencil in all positions (Fig. 77).

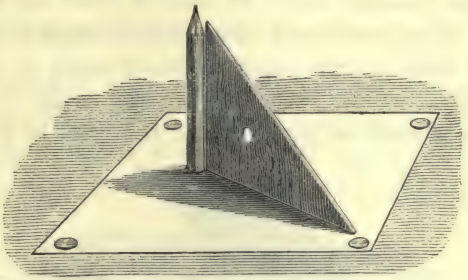


FIG. 77.

A builder uses a **plumb-line**, which is a leaden weight at the end of a string. When this hangs freely, it is said to be **vertical**, and it is at right angles to all the directions given by a spirit-level (Fig. 78). A surface

FIG. 78.

is a **horizontal plane** when a spirit-level shows that it is horizontal in every direction. A straight line which is perpendicular to every direction in a plane surface is said to be **normal** to that surface. So a vertical line is normal to a horizontal plane.

8. Open your book so that its two covers are at right angles, and place your set square so as to test whether they are at right angles or not. How do you place it? Fold a sheet of paper and fit it with the set square so that the two folds are at right angles. Mark on the paper with pencil the edges of the set square.

9. The smallest angle of one of your set squares is 30°. How would you place the set square so that the two folds of your paper make an angle of 30°?

10. How would you measure the angle between two planes, such as the covers of the book in Fig. 79?

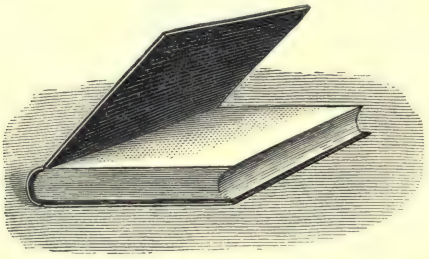


FIG. 79.

All figures drawn on a plane surface are called **plane figures**. You have now to learn something about the construction of various plane figures and their properties. This study is called **Plane Geometry**; while the study of the properties of figures which have thickness as well as length and breadth, figures like cubes and spheres which do not lie in one plane, is called **Solid Geometry**.

11. Draw figures to find out into how many regions three straight lines divide a plane. How many different cases are there?

CHAPTER X.

CONSTRUCTIONS. STRAIGHT LINES.

1. Take any piece of paper and fold it so as to make one edge, ACB, straight (Fig. 80 (i)). Fold it carefully in two so that A comes exactly on B (Fig. 80 (ii)). The line of fold CD is at **right angles** to the

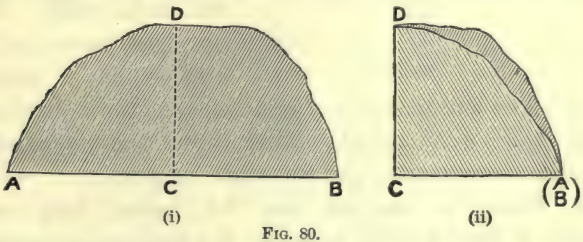


FIG. 80.

line AB, and the point C is the middle point of AB. The line CD may be called the **right bisector** of the line AB.

2. **To bisect a given straight line, i.e. to divide it into two equal parts.**

In Fig. 81 AB is any straight line. The two curved lines in the figure are parts of circles whose centres are A and B. The two circles have the same radius, which may be any length, provided it be greater than a certain length. What length? The two circles

intersect in the points C and D. These points are carefully joined by a straight line CD, which cuts AB at E. E is the middle point of AB, and the angles at E are right angles. Test these statements by measuring with your dividers and set squares. Then draw a straight line 3·7 inches long and bisect it in the same way. Do not draw more of the circle than is necessary. Test the accuracy of your construction. Measure AE, BE, $\angle AEC$, $\angle BEC$. Then repeat the experiment with unequal circles and see what happens.

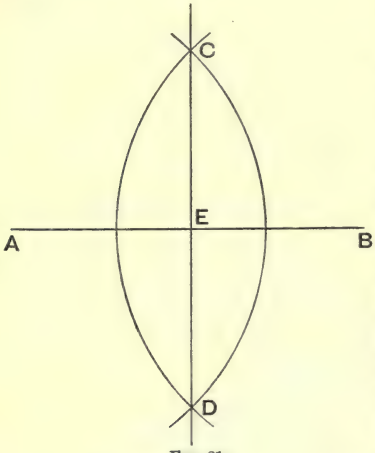


FIG. 81.

3. Draw a straight line 5·3 inches long and divide it into 4 equal parts by the same construction. Measure each part with your dividers. What is the measurement (two places of decimals)?

4. Draw any two straight lines obliquely on your paper like AB and XY in Fig. 82, and bisect them by the above construction.

5. The line CD in Fig. 81 bisects AB at right angles. When we say **bisect at right angles** we mean **use the construction given in Ex. 2.** Now draw any **triangle** ABC, and bisect each of the sides at right angles. The three **right-bisectors** ought all to meet in the same point.

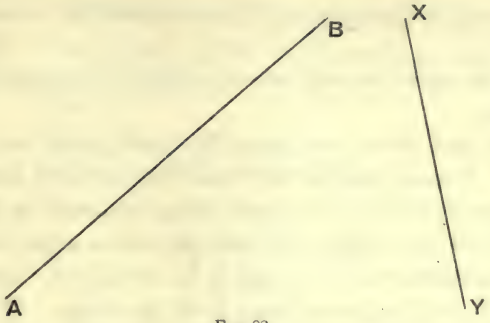


FIG. 82.

6. Draw any straight line AB (Fig. 83), and draw very carefully its right-bisector, as in Exercise 2. Now take any point P on this right-bisector, and

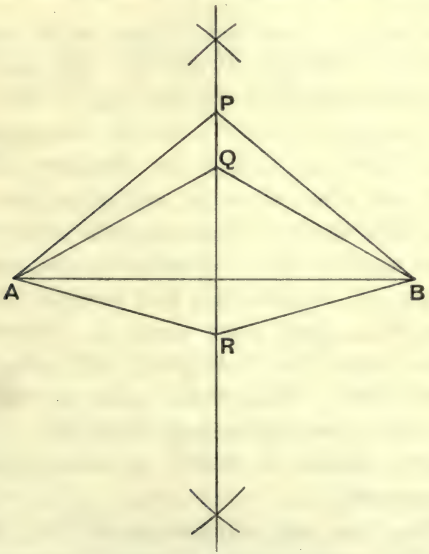


FIG. 83.

measure the distances of P from A and B. Are they equal?

Do the same for any other points Q, R, on the right-bisector.

7. A and B are two pegs 20 yards apart on a flat lawn. A man holds in his hand two rolls of measuring tape, the end of one tape being fastened to A and the end of the other to B, and he walks so as always to be **equidistant** from A and B, *i.e.* he allows the tapes to unroll, but he keeps them both the same. What is his path (or **locus**)? Draw a plan to illustrate your answer, scale 10 yards to the inch. When he is 20 yards from each peg, how far is he from the middle point of AB?

8. Now suppose there are 3 pegs not in a straight line with each other, A, B, and C. The man marks out his equidistant path for the pegs A and B, and again for the pegs B and C. Do the two paths intersect? Are there any points the distances of which from A, B, and C are all equal, and, if so, how many? Draw a careful figure to illustrate your answers.

9. Draw another figure illustrating the case when the three pegs A, B, and C are in a straight line.

10. Can you apply your result in Exercise 8 to finding the centre of a circle which will pass through three given points? Take three points at random and describe a circle passing through them all.

The centre should be found by construction and not by guesswork.

11. With your set squares as in Fig. 55 draw three parallel straight lines at equal distances. Take

a point A (Fig. 84) in the top line and from it rule straight lines ABC, ADE, AFG cutting the other

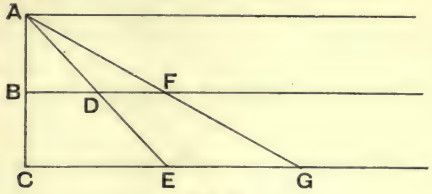


FIG. 84.

parallels. If your parallels have been accurately drawn, you should find on measuring that

$$AB = BC,$$
$$AD = DE,$$
$$AF = FG.$$

12. Draw five parallels at distance of 1 cm., and draw one straight line crossing them at right angles,

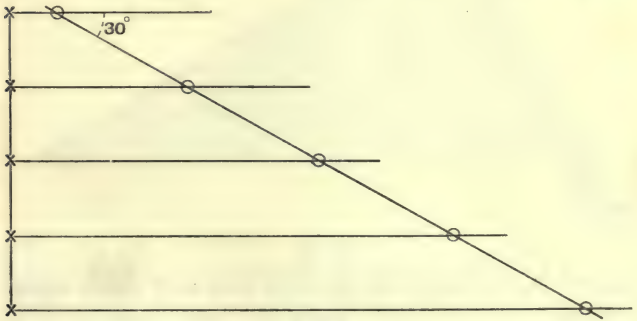


FIG. 85.

and another at an angle of 30° (use your protractor) as in Fig. 85. Measure the lengths of the portions x——x, o——o intercepted between the parallels.

E.G. F

13. You will now understand the following method of dividing a straight line into any number of equal

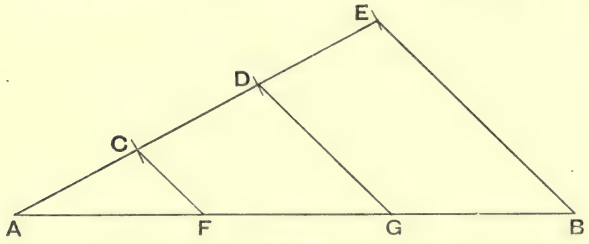


FIG. 86.

parts (say three). Suppose AB (Fig. 86) is the straight line. From A draw another straight line at

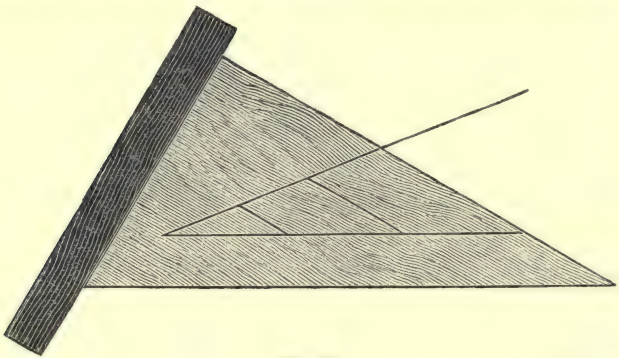


FIG. 87.

any angle, and step off on this line any three equal distances AC, CD, DE. Join EB, and through D, C draw DG, CF, parallel to EB. Then if AC, CD, DE were all equal, AF, FG, GB will be all equal.

Fig. 87 shows a convenient arrangement of the set square and straight edge for drawing the parallels.

In this way divide a straight line 4·3 inches long into three equal parts. Test your work by measuring each part to two places of decimals.

14. Divide a straight line 5·4 inches long into five equal parts.

15. Divide a line 5 inches long into seven equal parts.

16. Look back to Fig. 6, where an inch is shown divided into eighths and tenths. It is now possible

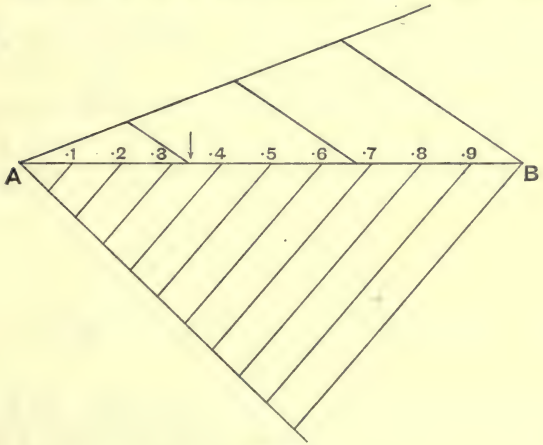


Fig. 88.

to find by construction the value of any fraction to two places of decimals. For instance, $\frac{1}{3} = 0{\cdot}33\ldots$ can be shown in this way. Take a straight line AB of any length and divide it into three equal parts, and also into ten equal parts (Fig. 88). It is plain that $\frac{1}{3}$ comes less than half-way between 0·3 and 0·4, and if you imagine this space divided into ten equal parts

(hundredths of AB) you will guess ↓ as coming nearly at the 3rd division.

Thus $\frac{1}{3} = 3$ tenths $+ 3$ hundredths $= 0{\cdot}33$.

Similarly, Fig. 88 shows $\frac{2}{3} = 0{\cdot}67$ nearly.

Now draw any straight line and divide it into seven equal parts by construction above the line, and into ten equal parts by a construction below the line. This will have to be done with great care. Then from your drawing find to two places of decimals the value of $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$.

17. **To divide a straight line into two parts which shall be in the proportion of 2 to 3.**

If the parts are 2 and 3, then the whole line must be 5. So if we divide the line into five equal parts and take the second division, there will be two parts

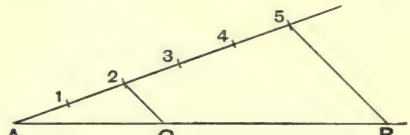


FIG. 89.

on one side of it and three on the other. It is not necessary in this case to draw all the parallels. Fig. 89 shows all that is wanted. AB is the line, and C is the point required. Now divide a straight line 3 inches long in the **ratio 2 : 3** (this means the same as **in the proportion of 2 to 3**). Measure the two parts.

18. Divide a straight line 5 inches long in the ratio of 3 to 4. Measure the two parts.

19. Divide a straight line 4 inches long in the ratio 13 : 19. It is not necessary to step off $13 + 19 (= 32)$

equal distances along your slanting line. You need only step off 1·3 inches (13 tenths) and 1·9 inches (19 tenths), taking the distances from your ruler.

FIG. 90.

Fig. 90 shows the arrangement. Measure the two parts.

20. In Fig. 91 is shown a line AB divided into twelve equal parts; at the same time it is divided

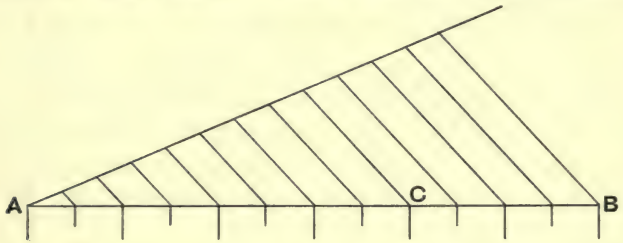


FIG. 91.

into 3, and into 6, the thirds and the sixths being marked by long and short upright lines. It is plain that

$$\frac{AC}{AB}=\frac{2}{3}=\frac{4}{6}=\frac{8}{12}.$$

Therefore, these three fractions must all be of the same value. Draw a similar figure to illustrate the fact that $\frac{2}{3}=\frac{6}{9}$.

21. By dividing a straight line into 10 parts and also into 16, illustrate the fact that $\frac{3}{8}=\frac{6}{16}=0{\cdot}37\ldots$ nearly.

22. Smith and Jones started a business with a capital of £500, Smith supplying £320 and Jones £180. Their profits for the 1st year were £57. 10s. 0d.

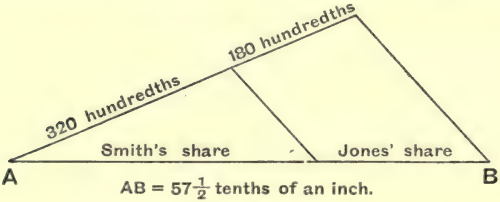


FIG. 92.

How much should each receive? (This is to be done by a geometrical construction. Fig. 92 gives you a hint of how to do it.)

23. Brown, Jones, and Robinson have to divide £87 between them in the proportion of their contributions which were £100, £150, and £200 respectively. Solve this problem by a geometrical construction. You ought to obtain answers correct to the nearest pound. Use the centimetre and millimetre scale for your measurements.

ADDITIONAL EXERCISES.

X. a.

1. Divide a line 5·8 in. long into 8 equal parts by repeated bisections.

2. From any point *O* in the circumference of a circle, draw three chords to any three points in the circumference; bisect these chords and on each of them as diameter draw a circle (each circle of course passing through *O*); these three circles will intersect in three points (not counting *O*), which, if the figure is drawn correctly, will be in the same straight line.

For the meaning of the word *chord* see IV. 5. (K.)

3. Draw any *quadrilateral*, or four-sided figure with unequal sides, and describe semicircles on the sides as diameters. [To find the centre of each semicircle you must bisect each side.] (K.)

4. Find the locus of the middle points of all straight lines drawn from a given point to meet a given straight line.

The following Exercises are tests for correctness in drawing:

5. Take two points A, B, and draw any number of lines in any directions through A, and two lines only through B, cutting all those which were drawn through A. The pair of lines through B will therefore form a four-sided figure with every pair that can be chosen out of those through A, and the opposite corners of every such four-sided figure should be joined, giving the diagonals of them all. The diagonals of each figure intersect in one point, giving as many new points as there are four-sided figures, all which points, if the figure be correctly drawn, will lie in the same straight line, which straight line will pass through B.

6. Take any two points A, B—it is required to find the mid-point of AB without drawing that line or using a straight edge. With centre A and radius AB describe a circle. On this circle cut off BC, CD, DE so that each chord equals AB. With centre B and radius BA describe a circle, and also another with centre E and radius EB. Let the last two circles cut in P, p. On the circle whose centre is B, starting from p measure chords equal to BA three times, cutting off arcs px, xy, yz. With centre P and radius PB describe a circle, from which cut off BM towards A so that chord $BM = Pz$. M is the mid-point of AB. (From Mascheroni's *Géométrie du Compas.*)

CHAPTER XI.

CONSTRUCTION OF PERPENDICULARS. DISTANCE OF A POINT FROM A STRAIGHT LINE.

1. You have seen how to draw perpendiculars to a straight line from a given point by means of your set square and rule (VI. 11). This is the usual way of drawing perpendiculars; but it is possible, and sometimes necessary, to use a construction which requires

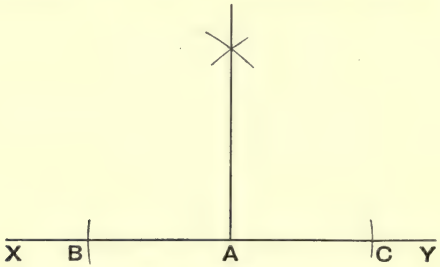


Fig. 93.

only compasses and ruler. For instance, we have **to draw from the point** A (Fig. 93) **in** XY **a straight line perpendicular to** XY. With the compasses mark off B and C equidistant from A. Then widen the compasses and describe circles (or arcs of circles) with centres at B and C, taking care that the two circles have the same radius. Join the point where the two

arcs intersect to A. This will be the required perpendicular. Would you get the same line if you took the point of intersection of the circles below XY? Try this for yourself, and compare with the construction for bisecting a line (X. 2).

2. **To draw a perpendicular to a straight line XY from the point X (Fig. 94) at one end of the line.** All that is necessary for this special case is to produce the line in the direction YX. The rest of the construction is the same as in Exercise 1. Make the construction and test your work with the protractor.

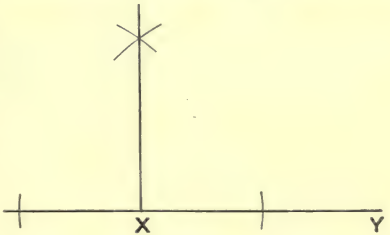


FIG. 94.

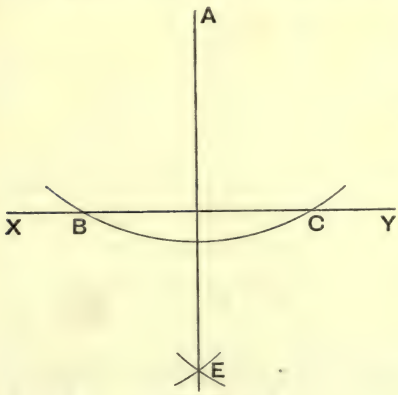


FIG. 95.

3. **To draw a perpendicular to a straight line XY from a point A outside.** First obtain the points B and C (Fig. 95). Then, taking these as centres, obtain the point E. Join AE. This is the required perpendicular. Make the construction for yourself and test its accuracy with set square and ruler.

4. Draw AB 2 inches long. At A draw AC perpendicular to AB and 2 inches long. At C draw CD perpendicular to AC and 2 inches long. At D draw DE perpendicular to CD and 2 inches long. Does E coincide with A? (All your constructions to be done by the method in Ex. 2.)

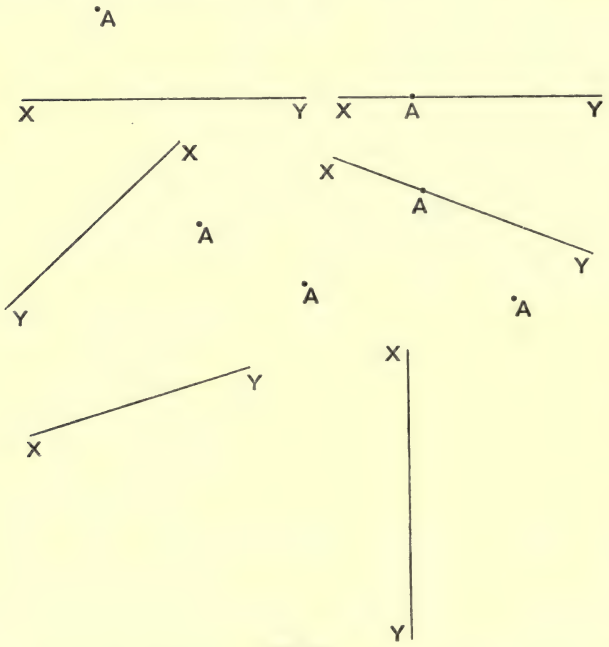


FIG. 96.

5. Draw any triangle with fairly large sides, and from each corner draw (by the method of Ex. 3) a perpendicular to the opposite side. Do the three perpendiculars all meet in one point?

6. It is important for you to be able to draw perpendiculars correctly with the free hand. In the following examples you are to draw in this way (Fig. 96) perpendiculars from the point A to the line XY, taking the line and the point as nearly as possible in the same position on your paper as they are in the book.

7. From A (Fig. 97) a perpendicular AB is drawn to XY. A circle is described with centre A and

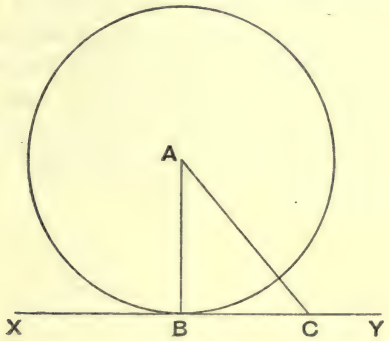


FIG. 97.

radius AB. This circle meets XY at B, but at all other points it is above the line. Therefore it meets the line without crossing or cutting it, and it is said to **touch** the line, and is called a **tangent.** If any other straight line is drawn from A to XY, such as AC, it is plain from the figure that AC is longer than the radius of the circle: and therefore AB is the shortest distance from A to XY. When we speak of **the distance of a point from a straight line,** we mean the shortest distance (VI. 15), and that is the

perpendicular distance. Now try if you can by construction obtain two points each distant 1 inch from a given straight line, and 2 inches from each other.

8. A man walks in a field by the side of a straight road, always keeping 20 yards away from the road. What is his **locus**? Draw a figure showing it, scale 10 yards to the inch.

9. Two roads meet at an angle of $\cdot 70°$. A man walks in the field, keeping at a distance of 20 yards from one road, and another man walks at a distance of 10 yards from the other road. Find where their two paths (or **loci**) cross. (Scale as in Ex. 8.)

10. Two straight lines cross at an angle of 73°. Find a point 1 inch away from each of them. How many such points are there? (Use only protractor, ruler and set square.)

CHAPTER XII.

ANGLES. CONSTRUCTIONS.

1. Make an angle of 70°, using your protractor. Cut out carefully the straight lines *BA*, *BC* which

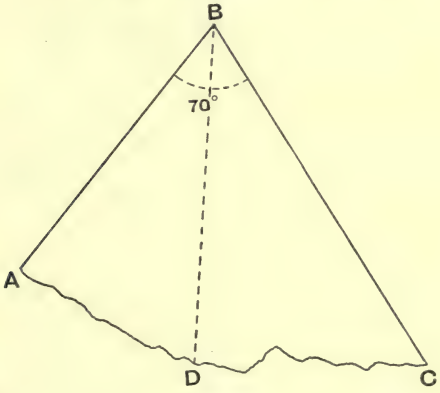


FIG. 98.

form the arms of the angle; the edge of the paper *AC* opposite to the angle may be torn off (Fig. 98).

Now fold the paper carefully so that the edge *BC* lies along the edge *BA*. The dotted line *BD* in the figure is meant to represent the line of the fold. Now

measure the angles *ABD, CBD.* They should each of them be 35°. The angle *ABC* is **bisected** by the line *BD,* because the two angles *ABD, CBD* are equal.

2. In the same way, cut out and bisect angles of 80° and 110°. In each case, before you make the fold, make a pencil mark on the edge of the paper where you think the bisector will come. Your guesses will probably be wrong at first, for it is harder to bisect an angle than a line by eye.

3. It is very necessary to be able to bisect an angle accurately by **construction**, *i.e.* with compasses and ruler alone.

BAC is any angle (Fig. 99). With the compasses, and any radius (not too small), mark off the two points

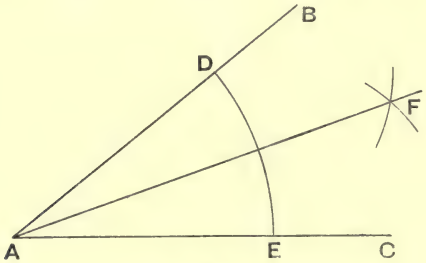


FIG. 99.

D and *E* equidistant from *A*. Now with *D* and *E* as centres describe two arcs of circles of equal radius, intersecting at *F*. (To make certain that these arcs shall meet, you may make their radius equal to the distance from *D* to *E*.) Now very carefully join *AF*. The angles *BAF, CAF* will be found to be equal. Also the two arcs into which *DE* is divided by the

line AF will be found to be equal. In this way bisect an angle of 58°, and measure with the protractor to see if your construction is accurate.

Remember that for accurate constructions

(i) Your lines must be fine;

(ii) Your circles must have their centres exactly at the right points;

(iii) In joining two points, you must take the greatest care to make the pencil pass exactly through the points before you actually rule the line (III. 8).

(iv) When a point is determined by the intersection of two arcs, it is well that these should intersect as nearly as possible at right angles to each other.

Now make an angle of 135° with your protractor and divide it into four equal angles. Measure the parts with your protractor.

FIG. 100.

4. Make various angles in the positions shown in Fig. 100. Make a dot with your pencil in each case to mark a point through which you think the bisector will pass. Then bisect each angle by construction.

5. In Fig. 101, AB is any straight line, and the angle DCB is 40°. What is the angle DCA? CE bisects the angle DCE, and CF bisects the angle DCF. Write

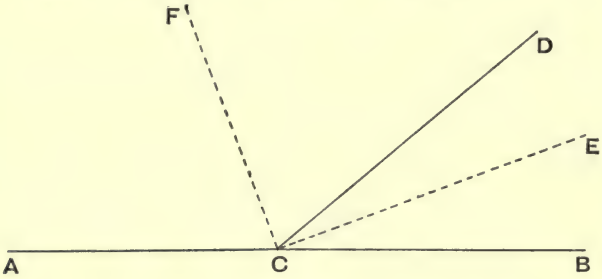


FIG. 101.

down the number of degrees in each of the angles DCE, DCF, ECF.

Answer the same question for Fig. 102, in which the angle DCB is 70°.

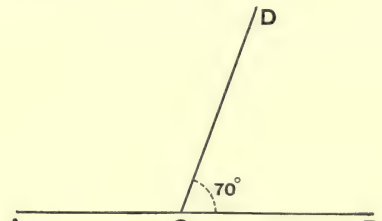


FIG. 102.

6. When a straight line DC stands on another AB, the two angles are called **adjacent** (=next-door neighbours). What is the sum of two adjacent angles of this kind? What is the angle between their bisectors? Can you give any reason for your second answer?

7. When two straight lines cross, as in Fig. 103, the two angles marked o are called **vertically opposite**; so are the two angles marked x. The top left-hand angle is 35°. What is each of the other angles? Can you discover any fact which is true of all vertically opposite angles?

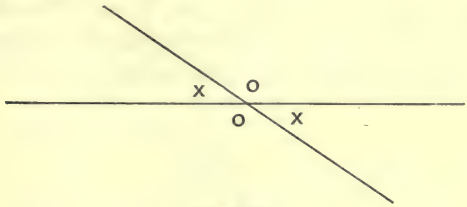


FIG. 103.

8. How many degrees are there in the sum of all the angles shown in Fig. 103?

9. Rule two straight lines crossing each other at any angle, and bisect each of the four angles formed. Only two constructions should be necessary. Why? Measure the angles between the bisectors.

10. Describe a circle of radius 3 inches, and draw two diameters at right angles to one another, using your set squares (as in VI. 11) to obtain the right angle. Now bisect each of the four right angles. Measure your eight angles, which should each be 45°, and measure the chords of the eight **arcs**, *i.e.* the eight parts into which the circumference of the circle is divided.

11. Obtain by construction (*i.e.* with ruler and compasses only), angles of 45°, 135°, $22\frac{1}{2}$°, $67\frac{1}{2}$° $112\frac{1}{2}$°, $157\frac{1}{2}$°, 270°, 315°, 330°. (All these angles should be

made in one figure, and should be tested with the protractor.)

12. Take any angle *ABC* and bisect it by the straight line *BD* (Fig. 104). Now take any point *P* on *BD* and draw from it *PM*, *PN* perpendicular to *BA*, *BC* (XI. 3). Measure *PM*, *PN*. They should be equal. Take any other point *Q* on *BD* and draw *QR*, *QS* perpendiculars, this time using the set squares. Measure *QR* and *QS*. Are they equal?

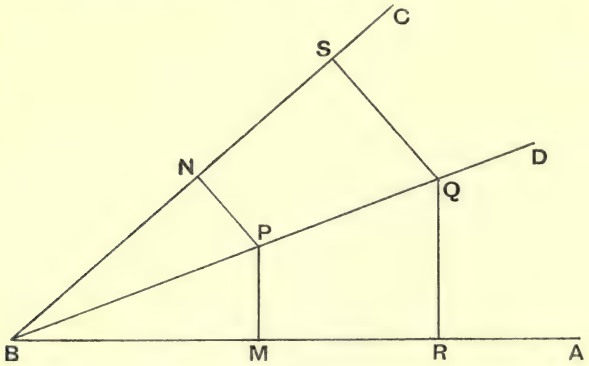


FIG. 104.

13. Two straight roads meet at an angle of 50°. A man walks in the field between them, so as always to be equidistant from the two roads. What is his locus? Draw a figure to illustrate your answer.

14. How many circles can be drawn as in Fig. 105 touching each of the two straight lines? What is the **locus** of their centres, that is, the line on which all the centres lie? It is possible to draw a circle touching each of three given straight lines. Copy

Fig. 106 which shows how to find the centre. When you have found the centre, be careful to draw a

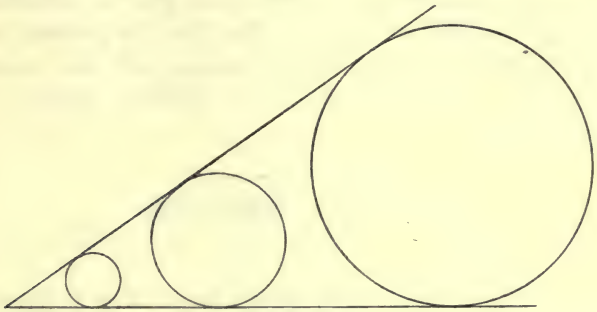


FIG. 105.

perpendicular from it to one of the lines, so as to get the exact radius of the circle.

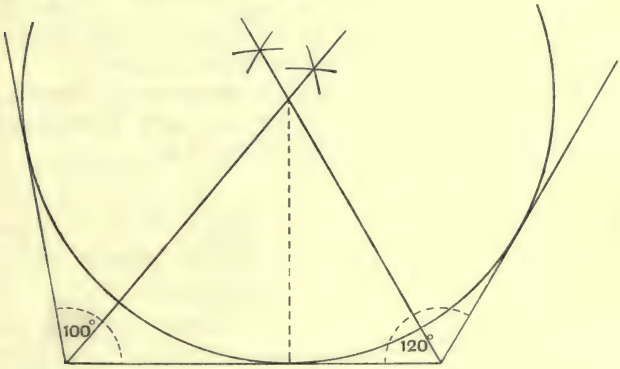


FIG. 106.

15. Draw any triangle and find the centre of a circle which will touch all its three sides. As before, the centre will be the intersection of two loci.

16. A triangle has three angles, and any two of these can be bisected, to give you a centre for the circle touching its sides. Does each pair of bisectors give you a different centre? Try this for yourself by drawing any triangle and bisecting all its angles carefully. Do you get three different points of intersection?

ADDITIONAL EXERCISES.

XII. a.

1. Draw a line CD meeting another line ACB in C, but not at right angles; bisect the angles ACD, BCD; and show that the bisectors are at right angles to one another. (K.)

2. Draw AB, AC 4 in. long inclined at an angle of 50°. Bisect the angle BAC by the straight line AD. Through A draw AE at right angles to AB and 1·5 in. long. Through E draw ED parallel to AB to meet AD at D. From D draw DF perpendicular to AC. Measure DF and the angle EDF.

3. Draw AB 4 in. and draw AC at right angles. Bisect the angle BAC by AD making AD 2·3 in. From D draw DE perpendicular to AB and measure DE and EA.

CHAPTER XIII.

PARALLEL STRAIGHT LINES ANGLES OF A TRIANGLE.

1. Draw two parallel straight lines at any distance apart, and draw a straight line crossing them at an angle of 47°. Measure, or write down without measuring, the number of degrees in each of the other seven angles in the figure.

2. The straight line cutting two parallels is sometimes called the **transversal**. In Fig. 107 the eight angles are numbered. The angles 1, 2, 7, 8 are called **exterior** angles (being outside the two parallels), and the other four are called **interior** angles. Angles situated like those of a **Z** as are called **alternate** angles. In Fig. 107 name

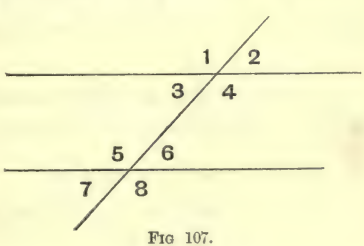


FIG 107.

(i) two pairs of alternate angles.

(ii) the interior angle adjacent to 2 (XII. 6).

(iii) the interior angle opposite to 2.

(iv) two interior angles on the same side of the transversal.

(v) two pairs of vertically opposite angles.

3. Two angles which when added together make 180° (or two right angles) are called **supplementary**. In Fig. 108 the lines AB and CD are parallel. EF is any transversal. Name four pairs of supplementary angles and two pairs of equal angles in Fig. 108.

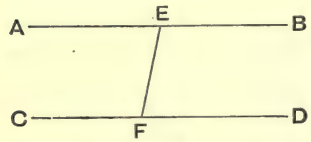


FIG. 108.

4. Name the supplementary pairs, and all the equal pairs in Fig. 107 (Ex. 2).

5. Cut a strip of paper with parallel edges, and cut it across at any angle, the line of the cut being like EF in Fig. 108. Then by placing the two pieces against one another, find whether the alternate angles are equal or not.

6. When two parallels are cut by a transversal, what connection exists between

(i) the alternate angles?

(ii) the interior angles on the same side of the transversal?

(iii) an exterior angle and the interior angle opposite to it on the same side of the transversal? (These are sometimes called **corresponding angles.**)

Illustrate your answer by a figure with letters.

7. A **triangle** is a plane figure bounded by three

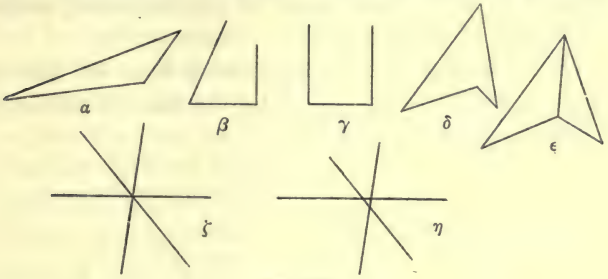


FIG. 109.

straight lines. State which of the diagrams in Fig. 109 are triangles, giving your reasons.

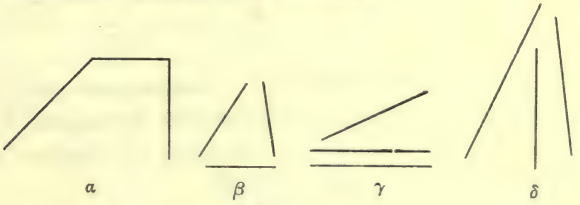


FIG. 110.

8. In Fig. 110 are shown four sets of three straight lines. State which of the sets are capable of forming triangles when produced, giving your reasons.

9. In Fig. 111 the angles are marked in degrees. Will the two slanting lines meet if produced? Give a reason for your answer.

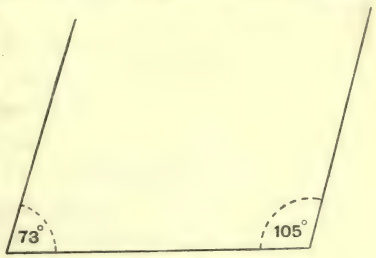


FIG. 111.

10. In Fig. 112 ABC is a triangle and AD is drawn parallel to BC. Name a pair of supplementary angles, and a pair of equal angles.

11. If in Fig. 112 the angle at B were 60°, and the angle at C 30°, what would be the size of the angle CAD? of DAB? of CAB?

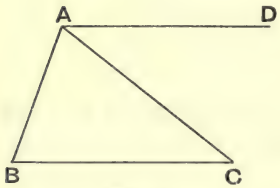


Fig. 112.

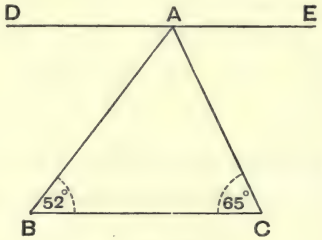


Fig. 113.

12. In Fig. 113 ABC is a triangle and DAE is parallel to BC. Two of the angles are marked in degrees. Copy the figure, marking in degrees the values of each of the angles at A.

13. If you know two angles of a triangle, can you find the third? What relation exists between them?

14. Draw four triangles of different shapes and measure carefully their angles with your protractor. What is the sum of the angles of each triangle?

15. Two angles of a triangle are 30° and 70°. What is the third angle?

16. Is it possible to make a triangle with its angles

(i) 45°, 45°, 90°? (ii) 70°, 80°, 29°?

(iii) 120°, 50°, 10°?

Give your reasons.

17. In Fig. 114 ABC is a triangle and the side BC is produced. ACD is called an **exterior angle**. Measure it. Measure the interior angles. Can you discover any connection between the number of degrees in ACD and the number of degrees in ABC, BAC? Draw four other triangles of different shapes, and measure in each case an exterior angle and the two **interior opposite angles**.

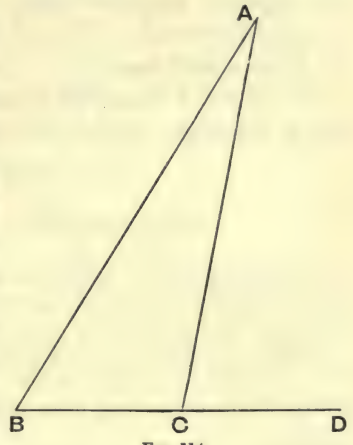


FIG. 114.

18. A yacht (Fig. 115) sails in a triangular course

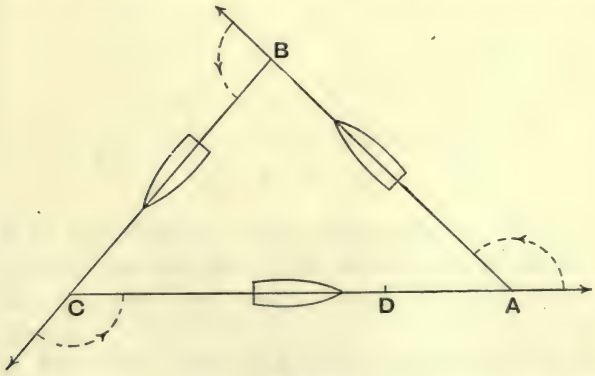


FIG. 115.

ABC, starting from the point D and coming back to

it. The angles that it turns through are shown in the figure. Measure them. Has the yacht turned completely round? What do the three angles amount to when added together?

19. Draw 4 triangles of any shape. Produce their sides, following round the triangle in one direction

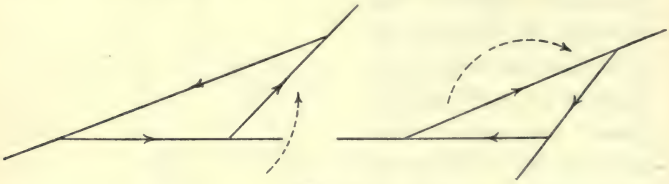


FIG. 116.

(Fig. 116), and find the sum of the three exterior angles.

CHAPTER XIV.

TRIANGLES: CONSTRUCTION.

1. *ABC* (Fig. 117) is a triangle. Measure its three sides and measure its three angles. From these measurements you are required to construct another triangle of exactly the same size and shape. Do

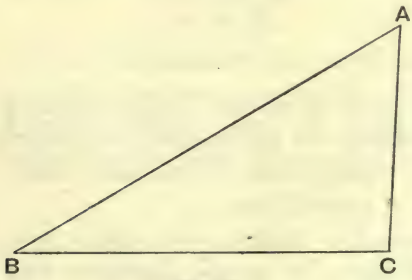


FIG. 117.

this in two different ways, stating as clearly as you can how you do it. Test with tracing paper if your triangles are really equal to *ABC* in all respects.

2. The three sides of a triangle are 3, 4, 5 inches respectively. Draw a straight line 5 inches long, and with your compasses draw arcs of circles of 3 in. and

4 in. radius respectively, as in Fig. 118. In how many points do these circles intersect? Which will you take for your triangle?

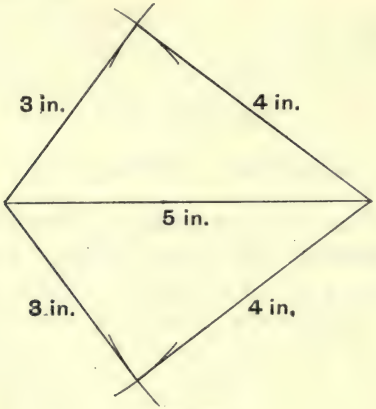


FIG. 118.

3. Copy your figure (Ex. 2) on tracing paper, and then by folding the paper along the 5-inch line, see if the two triangles are of the same size. Measure the angles of each of them.

4. Construct a triangle with sides 3, 4, 5 centimetres. Measure its angles. In what way does the triangle resemble that in Ex. 2?

5. Using your protractor, construct a triangle with angles 40°, 60°, 80°. Draw several such triangles. Must they all be equal?

6. It is impossible to make a triangle with its angles 40°, 60°, 81°. Why?

7. Fig. 119 shows a number of triangles AB_1C_1, AB_2C_2, etc. B_1C_1, B_2C_2, B_3C_3, etc., are all parallel. The

angle B_1AC_1 is 50°. B_1C_1A is 95°. What is the angle AB_1C_1? What are the angles of the other triangles?

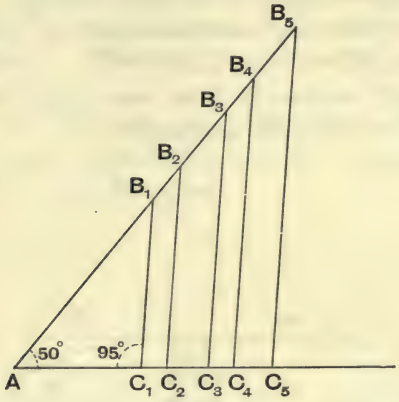


FIG. 119.

8. Fig. 120 is a sketch (not accurately drawn) of a triangle, and the measurements of two of the sides and of the angle between them are marked. Construct a triangle having these measurements.

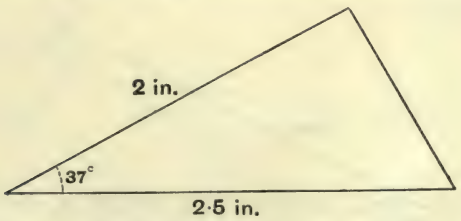


FIG. 120.

9. Construct on tracing-paper another triangle with the same measurements as in Ex. 8. When you place it over the former drawing, can you make

the two coincide? Measure the remaining side and the two remaining angles.

10. Make a rough sketch of a triangle ABC, and mark the following measurements on your sketch, $AB = 3$ in., $BC = 2{\cdot}7$ in., $\angle ABC = 70°$. Now construct the triangle accurately from your rough sketch, and measure the other side and angles.

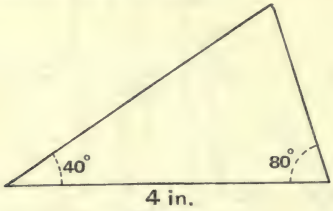


FIG. 121.

11. Construct a triangle from the rough sketch in Fig. 121.

Measure the other sides and angle. Construct another on tracing-paper from the same measurements, and see if it can be made to coincide with the former drawing.

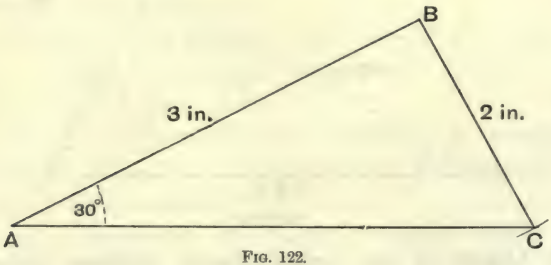


FIG. 122.

12. In constructing a triangle from the sketch in Fig. 122, first make the angle at A. Then mark

off the length AB. Then with B as centre and 2 in. radius, describe a portion of a circle which will give you the point C. Is there any different triangle which has the same three measurements? If so, draw it. Measure the remaining side and angles.

13. In Fig. 123 are shown 2 different triangles having the same three measurements. Construct one figure to show both triangles.

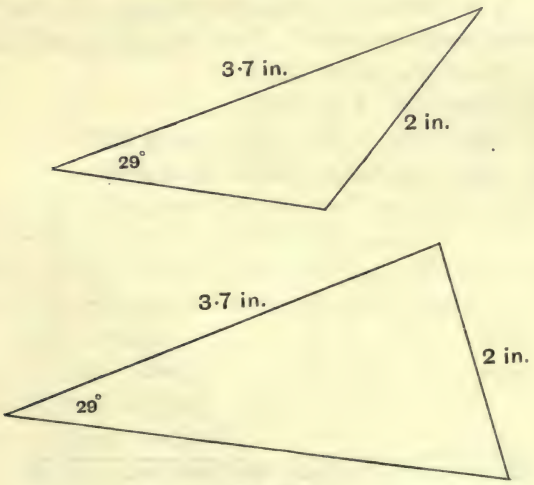


FIG. 123.

14. Make a sketch of a triangle ABC and mark the following measurements on it: $AB = 3{\cdot}3$ in., $BC = 2{\cdot}7$ in., $\angle BAC = 41°$. Now construct the triangle. Is there more than one value for the remaining side? Measure it.

15. Construct a triangle from the sketch in Fig. 124.

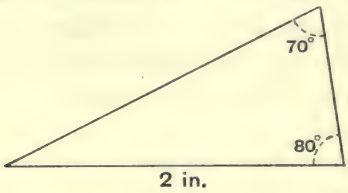


FIG. 124.

16. Is it possible to construct triangles having the following measurements? Give reasons in each case:

(i) Sides 4 in., 3 in., 1·2 in.

(ii) Sides 4 in., 3 in., 0·5 in.

(iii) $AB = 2$ in., $\angle$ at A 70°, $\angle$ at B 110°.

(iv) $AB = 2$ in., $BC = 1$ in., $\angle$ at $A = 40°$.

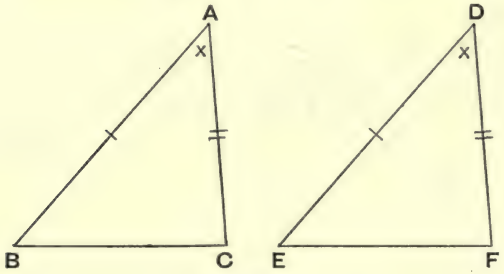


FIG. 125.

17. In Fig. 125 the sides and angles which are marked alike are equal. If you made a tracing of ABC and put it on DEF, the tracing would coincide with DEF. It is possible to reason this out without actually making the tracing. You will argue:

I know that if I make A come on D, and AB lie along DE, B will come on E. Why?

AC will lie on DF. Why?

C will come on F. Why?

The line BC will lie on EF. Why?

18. In Fig. 126 the angles and sides marked alike

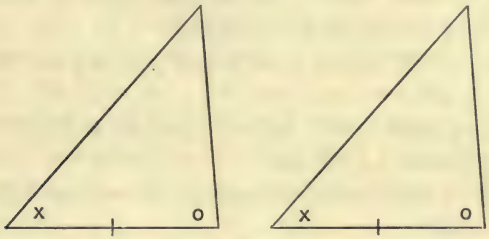


FIG. 126.

are equal. Can you give reasons for saying that the two triangles can be made to coincide?

19. To construct a triangle you must know some of its measurements. Is it enough to know

(i) Two sides?

(ii) Three sides?

(iii) Two angles?

(iv) Three angles?

(v) Two sides and the angle between them?

(vi) Two sides and an angle opposite to one of them?

(vii) Two angles and a side?

20. Construct a triangle equal to ABC (Fig. 125) by measuring the marked sides and angle? State how you make your construction.

21. Construct a triangle equal to one of those in

Fig. 126 by measuring the marked angles and side. Explain your method of construction.

22. Construct triangles from the following measurements:

(i) $AB = 5$ cm., $BC = 3$ cm., $\angle ABC = 63°$. Measure AC.

(ii) $\angle ABC = 70°$, $\angle ACB = 50°$, $BC = 2$ in. Measure AC and $\angle BAC$.

(iii) $\angle ABC = 70°$, $\angle ACB = 50°$, $AC = 3$ in. Measure BC.

(iv) $\angle ABC = 30°$, $AB = 3{\cdot}5$ in., $AC = 2{\cdot}7$ in. Measure $\angle ACB$, BC.

(v) $\angle ABC = 90°$, $AB = 3$ in., $AC = 5$ in. Measure BC.

23. ABC (Fig. 127) is any angle. To make an angle at D equal to it without using the protractor,

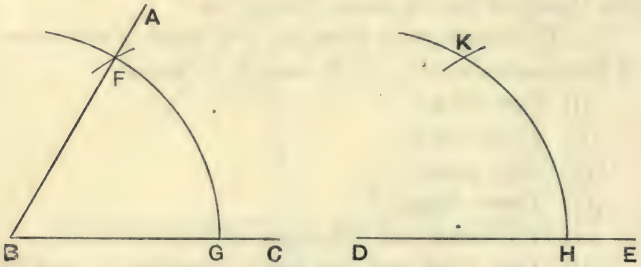


FIG. 127.

take the compasses, and, with centre at B, describe an arc of a circle FG. Then without altering the distance between the points of the compasses, describe another arc of about the same size with its centre at D. Now alter the compasses so as to step off the distance GF,

then taking H as the centre, step off that distance along the second arc HK. What straight line remains to be drawn? Can you give any reason for the second angle's being equal to the first? Make any angle and make another equal to it by this construction. Measure both.

24. Describe a triangle with sides 2, 2·5, 3 inches; at the ends of a straight line 4 inches long make angles equal to the angles at the end of the shortest side of the triangle. Produce the lines to meet and measure the remaining sides of the triangle so formed. Are they connected in any way with the sides of the first triangle? The two triangles are said to be **similar**.

25. When a straight line cuts two parallels, certain angles are equal. Can you devise *two* methods for drawing a parallel to a straight line through a given point, using only the construction in Ex. 23, and not using set squares?

(For Additional Exercises, see XV. a.)

CHAPTER XV.

SPECIAL KINDS OF TRIANGLES.

In this and the following chapters, make a note of any geometrical truth that you may come upon, and make a separate list of such truths.

1. Construct a triangle with sides 3, 3, and $1\frac{1}{2}$ inches. Measure its angles.

2. Construct a triangle with sides 2, 2, and 3 inches. Measure its angles.

3. A triangle which has two equal sides is called **isosceles** (*Greek*, isos = equal, scelos = a leg, ∴ isosceles = having equal legs.) Draw any isosceles triangle and measure its angles.

4. Draw a triangle ABC with the side BC 2·5 inches long, and the angles at B and C each 50°. Measure AB, AC, and the angle at A.

5. Make a triangle ABC with BC 3 inches long, and the angles at B and C each 70°. Measure the angle at A, and the sides AB, AC.

6. In an isosceles triangle the side which is not equal to the others is sometimes called the **base**, and the angle opposite to it is called the **angle at the vertex**, or the **vertical angle**.

[When the triangle is not isosceles, any side may be called the base, the opposite angle being called the vertical angle.]

Construct an isosceles triangle with its vertical angle 30°, and the sides containing the vertical angle each 3 inches long. Measure the angles at the base, and the base itself.

7. Make a triangle with its vertical angle 30° and the sides containing it 3·5 inches, and 2·5 inches. Measure the base and the angles at the base. Are these angles equal? If not, which is the greater, the angle opposite to the side 3·5 inches long, or the other?

8. Construct a triangle with sides 3·5, 4, and 4·5 inches long. Guess in degrees the size of each angle. Then measure them with your protractor, to find how far out you have been. Which side is opposite to the greatest angle, and which to the least? A triangle with three unequal sides is called a **scalene** triangle.

9. Construct a triangle with each of its sides 3 inches long. (A triangle with three equal sides is called **equilateral.**) Which is the greatest angle and which the least? Guess the size of each angle. Your guess should be a certainty if you remember what you learned in Chap. XIII.

10. Make an equilateral triangle, side 4 inches. What are its angles?

11. The vertical angle of an isosceles triangle is 20°. What are the base angles?

12. The vertical angle of an isosceles triangle is 80°. What are the base angles?

Construct an isosceles triangle, vertical angle 80°, base 3 inches.

13. One of the base angles of an isosceles triangle is 40°. What is the other base angle? What is the vertical angle?

14. A triangle with one right angle is called a **right-angled triangle.** Is it possible to construct a right-angled isosceles triangle? If so what are its other angles?

15. Is it possible to construct a triangle with two of its angles right angles? Give your reasons.

16. What is an obtuse angle? (See V. 30.) What is meant by an **obtuse-angled triangle?** Construct any obtuse-angled isosceles triangle. How many obtuse angles has it? How many acute angles?

17. Make a triangle with all its angles acute. Such a triangle is called an **acute-angled triangle.**

18. Make a triangle on a base of 3 inches and with the angles at each end of the base 60°. Measure the other two sides.

19. In a triangle ABC, the side AB is equal to the side AC. Are two of the angles equal? If so, name the equal angles.

20. In a triangle ABC, the angle at B is equal to the angle at C. Are two of the sides equal? If so, name them.

21. In a triangle PQR, the angle PQR is equal to the angle QPR. Name the two equal sides.

22 In a triangle ABC, the side AB is the greatest, and the side AC the least of the three. Name the angles in order of magnitude.

23. In a triangle ABC, the angle at B is the greatest, and the angle at C the least of the three. Name the sides in order of magnitude.

24. The angles of a triangle are each of them 60°. What sort of a triangle is it?

25. You have probably found out, if you did not know before, that a triangle which has two equal sides has also two equal angles. If you are told the equal sides, how can you tell which are the equal angles? State the rule as clearly as you can.

ADDITIONAL EXERCISES.

XV. a.

1. With your instruments draw an equilateral triangle with sides 5 centimetres long.

Show (without proof) how to cut it into two equal right-angled triangles, and to fit these two triangles together again so as to make (1) a rectangle, (2) an isosceles triangle that is not equilateral.

2. Describe a triangle which shall have its sides $1\frac{1}{2}$ and 2 inches, and the angle between them a right angle. (K.)

3. Find a point 7 ft. distant from each of two points which are 8 ft. from one another, scale 4 feet = 1 inch. (K.)

4. Make a triangle with sides 4·5 in., 3·6 in., 2·7 in., and measure the three angles.

5. Describe a triangle with sides 3, 4, and 5 half-inches, and measure the angles.

6. Describe six equilateral triangles round a point, not overlapping one another, and having a common vertex at that point. These six triangles will make a regular hexagon. (K.)

7. Construct an isosceles triangle, base 3 in., and sides 2·5 in. Through each angular point draw by construction lines parallel to the opposite sides, and produce them to meet.

8. Make a triangle CAB with base AB 75 mm. and angles at A and B, 53° and 37°. From C draw CD perpendicular to AB and measure it.

9. Construct the triangle ABC, having AB 3·2 in., ABC 39°, BAC 53°. Construct a similar triangle with longest side 4 in.

10. Make a triangle with two sides 72 mm. and 45 mm. and contained angle 60°. Measure the third side and the angles.

11. Construct a triangle ABC, given AB 7·5 cm., BC 7·8 cm., AC 6·9 cm. Bisect AB and AC at D and E and measure DE.

12. Construct a triangle with sides 3 in., 2·8 in., 3·4 in. Draw a similar triangle with longest side 2·6 in. and measure the other sides.

13. Construct a triangle ABC, given AB 2·7 in., AC 3·2 in., angle BAC 57°. Measure BC, and bisect all the angles.

14. A man wishes to find the distance of a boat at anchor: he measures 200 yards on the beach, and, taking that line as base AB, and the boat as the point C, he finds the angle $CAB = 90°$, the angle $CBA = 60°$. Show that the boat is 346 yards from A. (K.)

15. A man wishing to find the distance of a fort measures a base of 100 yards, and finds that with the fort as the vertex or third point of a triangle, and the 100 yards as base, the angles at the base are each $67\frac{1}{2}°$. Find how far the fort is from either end of the base; and measure the third angle.

16. Describe an equilateral triangle, side two inches, and by means of it describe three circles, one inch in radius, each touching the other two. (K.)

17. A man 50 yards from the edge of a moat close round a castle finds that the castle tower subtends an angle of 45°, while at the edge of the moat it subtends 60°. Find the width of the moat. (K.)

18. Construct the triangle ABC, in which $AB = 3·3$ in., $AC = 2·2$ in., and $BAC = 90°$ (to be made by construction).

19. Construct a triangle, equiangular to ABC, whose greatest side shall be 4·2 in. Measure its smallest angle.

20. Draw a plan of a triangle which shall have one side, AB, 10 feet, the angle ABC 30°, and the side AC 7 feet. To

determine the point C it will be necessary to describe a circle with centre A and radius 7 feet. This will cut BC in two points (about $9\frac{1}{2}$ feet apart), either of which will suit for the third point of the triangle. (K.)

21. Construct an isosceles triangle, base $1\frac{1}{2}$ inches, the angles at the base being each double that at the vertex. (K.)

22. Construct a triangle having one of its sides AB equal to 3 inches, another AC equal to $3\frac{1}{4}$ inches, and the angle at B equal to 50°, and measure the remaining angles. (K.)

23. Construct a triangle the lengths of whose sides are $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$ inches respectively, and measure the angles. (K.)

24. Draw a plan of a triangle whose sides are 11 and 23 chains and included angle 45°, and show that the third side is nearly $17\frac{1}{3}$. (K.)

25. Suppose that you have taken the measurements of the sides of a triangular field, viz. 10 chains, 15 chains, and 20 chains, draw a plan of the field (scale 10 chains = 1 inch). (K.)

26. The direct distance from Coventry to Warwick is 10 miles, from Rugby to Coventry is $11\frac{1}{2}$ miles, and from Rugby to Warwick 15 miles; how far is Rugby from the middle point between Coventry and Warwick? (K.)

27. If one angle of a triangle be 20° greater than the second, and the third be a right angle, find the second. And draw the triangle, the side opposite the right angle being 3 inches, and measure the shortest side. (K.)

28. Upon a straight line 2 inches long, construct an isosceles triangle having a vertical angle of 35°.

29. Describe a triangle, having given the base 2 inches, the sum of the two sides $4\frac{1}{2}$ inches, and one angle at the base 55°.

30. Construct a triangle, with perimeter 7 in., and sides in ratio of 2 : 3 : 4. (**Perimeter** = sum of the sides.)

31. Construct an equilateral triangle with altitude 2 in.

32. Draw a triangle, given perimeter 5 inches, and two angles 47° and 63°, and describe a similar triangle with longest side 3 in.

CHAPTER XVI.

PROPERTIES OF TRIANGLES—(*Continued*).

1. Draw a large isosceles triangle and cut it out carefully. Then fold it in two so that the equal sides lie along one another. You will find, if you do your part correctly, that certain other lines and angles coincide. Draw a sketch plan of your triangle showing the line of fold, and marking the lines and angles which coincide.

2. Draw an isosceles triangle with sides 3, 3, and 2 inches. Bisect the vertical angle of your triangle, and make the bisecting line long enough to cut the base. Does it bisect the base? Test by measurement. Is it perpendicular to the base? Test this by your protractor.

3. Draw another isosceles triangle, and bisect the base (X. 2). Join the middle point of the base to the vertex. Does this line bisect the vertical angle? Are the angles it makes with the base right angles?

4. Cut out a paper triangle with sides 4, 5, and 6 inches. Fold it carefully so that the two shorter sides lie along one another. The line of fold is the bisector of the largest angle of the triangle. Does

the line of fold pass through the middle point of the third side? Is it at right angles to the third side?

5. Construct a triangle ABC, AB 2 inches, BC 3 inches, CA 4 inches. Then bisect the angle BAC. Produce this bisector to cut BC in the point D. Measure the lengths BD, DC. Measure the angles at D. Are they right angles?

6. Construct another triangle exactly as in the preceding exercise. Bisect the side BC at the point E. Join EA. Measure the angles BAE, CAE with your protractor. Are they equal? Measure the angles at E. Are they right angles?

7. Cut out a paper triangle with sides 4, 5, and 6 inches. Fold it as in Fig. 128 making the line

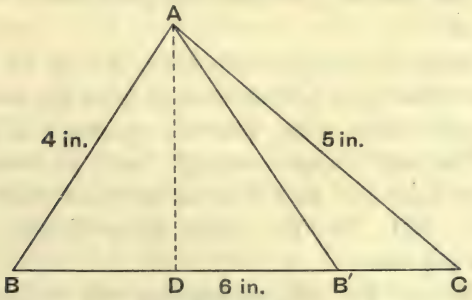


FIG. 128.

of fold AD pass through A, and making B fold over on the line BC in the position B'. If you do this accurately, your line of fold AD will be perpendicular to BC. Why? (V. 3.) Is D the middle point of BC?

8. Instead of cutting out, construct a triangle as in Ex. 5 and draw AD perpendicular by construction or by set squares (VI. 11, XI. 3). Measure BD, DC, and the angles BAD, DAC.

9. Take the paper triangle used in Ex. 7. You have folded it so as to get the perpendicular from A on BC. Now unfold it, and fold it again so as to get the perpendicular from B on AC, and the perpendicular from C on AB. The three lines of fold should be visible. Do they all meet in a point? They should do so if you have made your folds accurately. This point where the three perpendiculars meet is called the **orthocentre** of the triangle.

10. Repeat Ex. 9, only this time *draw* your triangle and obtain the three perpendiculars by construction.

11. Make a triangle with sides 3·5, 4, 4·5 inches, and draw the three perpendiculars from the corners to the opposite sides, using your set squares (VI. 11).

12. Cut out a paper triangle with its sides 3, 4, 5 inches long, and fold it so as to bisect each of the angles. (XII. 1.) The three lines of fold should meet in a point.

13. Repeat Ex. 12, constructing the triangle and bisecting the angles by construction. (XII. 3.) Do the three bisectors meet in a point? (Three or more lines which all pass through the same point are said to be **concurrent**.)

14. Construct a triangle with sides 4, 5, and 6 inches, and bisect the angles. From the **point of**

concurrence draw perpendiculars to each of the three sides of the triangle. Measure each of these perpendiculars to the nearest hundredth of an inch. Refer back to XII. 12, and give a reason for the three lines being equal to one another. A circle can be described with centre at the point of concurrence to pass through the feet of the three perpendiculars. Describe this circle. It is said to be **inscribed** in the triangle, and the centre of the inscribed circle, the point where the three bisectors of the angles meet, is sometimes called the **incentre** of the triangle.

15. Find the incentre of a triangle whose sides are 3·5, 4, 4·5 inches, and draw the inscribed circle. Measure its radius.

16. In X. 1, 2 you were told how to obtain the right bisector of a straight line by construction and by folding. Cut out a triangle with its sides 5, 6, 7 inches, and obtain by folding the right bisectors of the sides. Are the three lines of fold concurrent?

17. Construct a triangle with sides 3·7, 4, 4·3 inches, and draw the three right bisectors. The point of concurrence is equidistant from the angular points of the triangle (X. 10). Measure this distance, and describe the circle passing through these three points. This circle is called the **circumscribing circle**, and its centre is sometimes called the **circumcentre**. Note that to find the circumcentre, only two right bisectors need be drawn.

18. Find the circumcentres and draw the circumscribing circles of triangles having the following dimensions:

(i) Sides 3, 4, 5 inches.
(ii) „ 2, 3, 4 „
(iii) „ 4, 5, 6 „
(iv) „ 2·5, 6, 6·5 centimetres.

In each case measure the angles of the triangle, and note whether the circumcentre falls inside or outside the triangle.

19. Construct an isosceles triangle, sides 5, 5, 3 inches, and find its orthocentre, incentre, and circumcentre. All these points should lie in one straight line. What straight line?

20. Construct an equilateral triangle, side 4 inches, and find its orthocentre, incentre, and circumcentre. Draw the inscribed and circumscribing circles.

21. The straight line joining one angular point of a triangle to the middle point of the opposite side is called a **median**. Construct a triangle, sides 3, 4, 5 inches, and draw the medians. Are they concurrent?

22. Construct a triangle, sides 4, 5, 5·6 inches, and draw

(i) the perpendiculars from the angular points on the opposite sides.
(ii) the bisectors of the angles.
(iii) the right bisectors of the sides.
(iv) the medians.

23. A circle can be drawn touching three straight lines, even if they do not form a triangle, unless all three lines are parallel. (XII. 14.) Draw two straight lines parallel to one another, and 2 inches apart, and a third straight line crossing them at an

angle of 60°. Draw two circles touching all three lines. Show how the centres are found.

24. A circle touching one side of a triangle and the other two produced is called an **escribed** circle. Draw an equilateral triangle, side 1 inch, and draw the three escribed circles. Show the construction for finding the centres.

ADDITIONAL EXERCISES.

XVI. a.

1. Inscribe a circle in a triangle ABC, having $AB=4$ in., $AC=2$ in., and $BC=3$ in.

2. Construct a triangle with two sides 3 in. and 3·4 in. respectively and included angle 72°. Measure the third side and inscribe a circle.

3. Construct the triangle ABC, $AB=4{\cdot}4$ in., $AC=1{\cdot}8$ in., and $BAC=55°$. Describe the escribed circle which touches the shortest side and the other two produced.

4. Construct the triangle ABC, having $AB=4{\cdot}2$ in., $AC=5$ in., and $BC=3{\cdot}7$ in. Describe a circle about it.

5. Show (with proof) how to circumscribe a circle about a given triangle. With your instruments make a triangle having sides 6, 8, 12 centimetres long and circumscribe a circle about it.

6. Describe three circles $\frac{1}{2}$ inch in radius, each of which touches the other two, and about them describe an equilateral triangle, each side of which touches two of the circles; measure the side of the triangle.

7. Construct the triangle ABC, $AB=4{\cdot}2$ in., $AC=3{\cdot}1$ in., and $BC=1{\cdot}8$ in. About it describe a circle.

8. Describe a triangle of which two sides are 2·5 in. and 2 in., and the contained angle 120°, and about it circumscribe a circle.

9. Inscribe a circle in a sector of a circle of radius 3 in. and vertical angle 35°.

(A **sector** of a circle is a figure contained by an arc and the radii which join the centre to the ends of the arc. A circle inscribed in this must touch the arc at its middle point. Hence if we draw the radius which bisects the angle of the sector, and at the end of this radius draw a line at right angles to meet the other two radii, the problem becomes the same as the inscribing of a circle in the triangle so formed.)

10. Draw a circle of 2 inches radius, and in it place 4 circles, each of them touching the outer circle and two of the others.

Divide the circle into 4 equal sectors.

11. In a circle of diameter 5 inches inscribe 5 equal circles each touching the circumference, and the two adjacent circles. Show construction.

(See XVII. 23, for dividing the circle into 5 equal sectors.)

12. Draw an equilateral triangle, side 4 in. Divide it into four equilateral triangles, and about each of them circumscribe a circle.

13. Draw a circle of 2·5 in. radius, and in it inscribe six equal circles, each touching two of the others and the first circle.

14. Describe four equal circles to touch externally a circle of radius $\frac{1}{2}$ inch, and to touch each other.

15. About an equilateral triangle, side 1·6 in., describe three circles, each touching one side of the triangle and two others produced.

16. Describe an isosceles triangle with sides $2\frac{1}{2}$ in. and vertical angle 35°; and with the angular points as centres, describe three circles to touch each other.

17. In an equilateral triangle, 3·6 in. side, inscribe three equal circles, each touching 2 others and 2 sides of the triangle.

18. In an equilateral triangle, side 3·7 in., inscribe three equal circles, each touching the other two, and one side of the triangle.

CHAPTER XVII.

CONSTRUCTION OF SPECIAL ANGLES AND POLYGONS.

1. What are the angles of an equilateral triangle?

2. In Fig. 129 you are shown how to construct an angle of 60°. Construct it. Can you explain why the angle is 60°? Test the accuracy of your construction with your protractor.

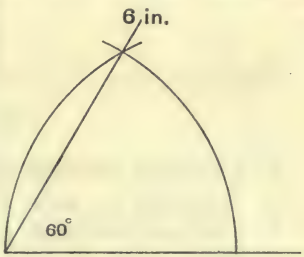


FIG. 129.

3. In making an angle of 60°, it is not necessary to describe complete circles, but only quite small arcs, provided their radius and centres are correctly taken. Make an angle of 60° as in Fig. 130.

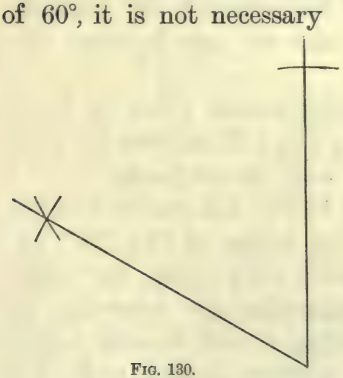


FIG. 130.

4. In Fig. 131 the construction for 90° (XI. 1) is given, also for 45° (by bisecting the angle of 90°). Make this construction. Test with

your protractor. There is one obtuse angle in the figure. Which is it? Measure it.

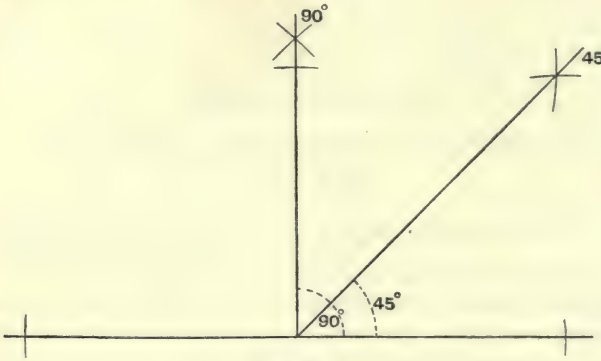


FIG. 131.

5. Construct an angle of $22\frac{1}{2}°$.

6. Construct an angle of $67\frac{1}{2}°$.

7. Construct an angle of 135°.

8. What angles can be obtained by bisecting the angles between 135° and 90°, also between 135° and 180°? Obtain them.

9. In Fig. 132, the constructions given in Ex. 3 and 4 are combined. Copy Fig. 132 and test the accuracy of your construction with the protractor.

10. Any of the angles in Fig. 132 can be bisected. Construct a figure showing angles of 15°, $22\frac{1}{2}°$, 33°, 45°, 60°, 75°, 90°, 105°, 120°, $127\frac{1}{2}°$, 135°, 150°, 165°.

11. There is no construction for trisecting an angle, that is to say dividing it into 3 equal angles. It can be done by trial however in this way. Let *XYZ* (Fig. 133) be any angle. With centre *Y* draw an

arc of a circle XZ. Take your dividers and adjust them to step off as nearly as possible a third of

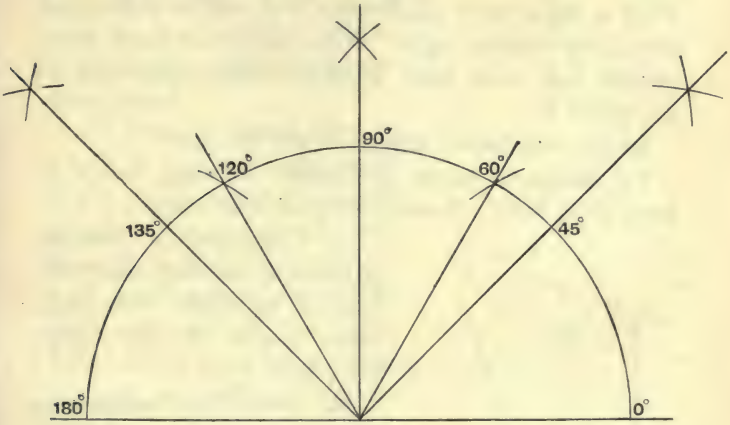


FIG. 132.

the arc ZX. Suppose your first attempt is ZA. Step off along the arc ZA, AB, BC. If your guess were

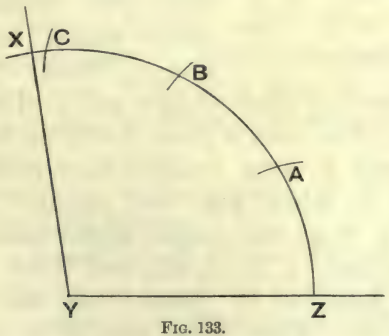


FIG. 133.

correct, C would be at X. If not, your error three times repeated is the distance CX. So lengthen

(or shorten) your step by as nearly as possible *one third* of the distance CX and step off from Z again. With a little more adjustment you will be able to get your step exactly right. Now mark the steps (they should not have been marked before) and join to the point Y.

In this way trisect angles of 30°, 60°, 120°.

12. Make a paper protractor showing every 5° from 0° to 180°. Number it from both ends.

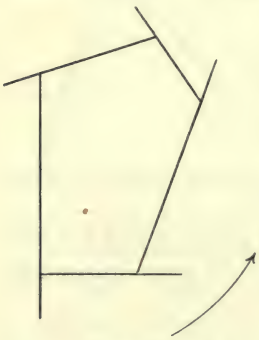


FIG. 134.

13. Construct a circular protractor, showing every 15° from 0° to 360°. Use it to make angles of 210°, 270°, 315°, 330°.

14. Draw any **polygon** (a figure of more than 4 sides). Produce the sides, going round the figure in one direction (Fig. 134). Measure all the outside angles. Add them together. What is their sum?

15. Draw any other polygon and produce the sides as before. Now take a point O outside the polygon, and through it draw straight lines parallel to the directions in which you have produced the sides. (Fig. 135.) Mark the outside angles (**exterior angles**) of the polygon, α, β, γ, etc., and mark the corresponding angles at O. What is the sum of all the angles at O? What is the sum of all the exterior angles of the polygon?

16. What is the sum of the exterior angles of a

quadrilateral, obtained as in Ex. 15 by producing the sides, going round in one direction as before? What is the sum of the exterior angles of a triangle obtained in the same way?

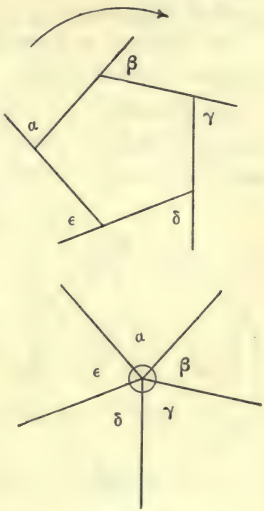


FIG. 135.

17. Draw a sketch plan of a field of any **rectilineal** shape (XIX. 11), showing the angles which a man would turn through in walking round the field, starting from the middle point of one of the sides. What angle does a man turn through in walking round a ring?

18. Draw any five-sided figure and produce the sides as before. At each corner there are two angles, an exterior and an interior angle. What is the sum of each pair of angles? What is the sum of all the pairs of angles? From this find the sum of all the interior angles by subtracting the sum of all the exterior angles.

19. Answer the same questions as in Ex. 18 for 6-, 7-, and 8-sided figures.

20. In a **regular** figure, all the sides are equal, and all the angles are equal. What is each exterior angle of a regular **pentagon**? What is each interior angle? (A pentagon is a five-sided figure.)

21. Answer the same questions as in Ex. 20 for

regular **hexagons**, **heptagons**, and **octagons** (6-, 7-, and 8-sided figures).

22. Obtain a formula for the exterior angle of a regular n-sided figure.

23. Inscribe a regular pentagon in a circle of radius 2 inches. Describe the circle, and at the centre make an angle equal to $\frac{1}{5}$ of four right angles. If you have plotted off your angle accurately, the chord AB of the circle (Fig. 136) ought to go exactly 5 times round the circumference when you step it off. Try this, without making marks until you are right. Remember that it is very important to make your steps exactly on the circumference of the circle, not inside or outside it. When you have got the distance accurately, you can mark the steps and join them to form a pentagon.

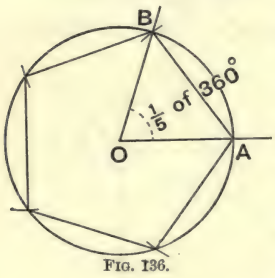


FIG. 136.

24. A regular hexagon is more easy to construct than a pentagon. (Ex. 3.) Inscribe a regular hexagon in a circle of 2 inches radius, obtaining the angle at the centre by construction.

25. Inscribe a regular heptagon in a circle of 3 inches radius. Also in another circle of 3 inches radius inscribe a regular octagon, obtaining the angle at the centre by construction.

26. Construct a regular pentagon on a base of 2 inches. Draw AB 2 inches long, and make the exterior angle CBD (Fig. 137) $\frac{1}{5}$ of four right angles.

Make $CB = BA$. Draw the right bisectors of AB, BC. These will intersect in O, which is the centre of the circle circumscribing the pentagon. Draw this circle, and the other corners of the pentagon can be stepped off on it.

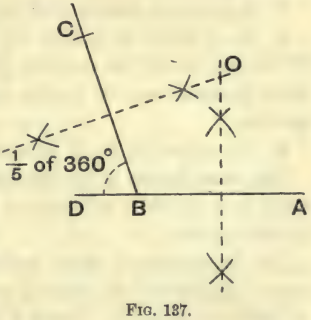


FIG. 137.

27. Describe a regular hexagon on a base of 2 inches. The circumscribing circle can be obtained far more easily than that of the pentagon.

28. Describe a regular octagon on a base of 1·5 inches.

29. Describe a regular heptagon, each side 2 inches.

ADDITIONAL EXERCISES.

XVII. a.

1. Draw ACB 4 in., and on it as diameter describe a semicircle. Draw the radii which make angles of 30°, 60°, 90°, 120° and 150°, respectively, with CB, obtaining the angles by construction. Divide the angle of 30° into three equal parts by trial to show 10° and 20°.

2. Construct a triangle base 3·3 in. and base angles 30° and 60°. Construct the angles, and escribe a circle to touch the shortest side. (XVI. 24.)

3. Construct angles of 165° and $22\frac{1}{2}°$ without using protractor. Show the method clearly.

4. At the point A in AB make angles of 30° and 15°, one above and the other below AB.

5. Construct angles of 20°, 75° and $112\frac{1}{2}°$.

6. Draw a rhombus (XVIII. 9) with side 2·5 in., and one angle 75° (to be made by construction), and in it inscribe a square (XIX. 1).

7. In a circle, radius 2·43 in., inscribe a regular pentagon.

8. In a circle of radius 2·7 in., inscribe a regular hexagon. With centres at the angular points and radii equal to half the sides describe arcs of circles outside the hexagon touching one another.

9. On a line 2·3 in. as base describe a regular hexagon.

10. Inscribe in a given circle an eight-sided figure which shall have its sides subtending alternately angles of 60° and 30° at the centre. (K.)

11. In a circle of $2\frac{1}{2}$ in. radius inscribe a five-rayed star, the rays to be formed by joining each corner of a regular pentagon to the next but one. Draw a second star with its rays parallel to and $\frac{1}{8}$ in. inside those of the first star.

12. Draw a regular pentagon with side 2·3 in., and in it inscribe 5 equal circles each touching 2 others and one side of the pentagon.

13. In a circle, radius 2·75 in., inscribe a regular heptagon.

14. Construct a regular nine-sided figure of $1\frac{1}{2}$ inch side, and inscribe a circle within it.

15. Inscribe *any* hexagon (with unequal sides) in a circle, and show by a figure that the intersections of the three pairs of opposite sides (viz. of the first and fourth, of the second and fifth, and of the third and sixth), when produced to meet, lie on the same straight line. (K.)

16. What is the sum of (*a*) the interior angles; (*b*) the exterior angles of a triangle? Deduce a formula for the magnitude of an angle of a regular polygon of n sides, and from this obtain the number of degrees in the angle of a regular pentagon.

17. Construct a hexagon, side 2 in. Divide it into three equal rhombi (XVIII. 9) and inscribe circles.

18. Describe an equilateral octagon in a circle whose radius is 1 inch.

CHAPTER XVIII.

AREAS OF PARALLELOGRAMS AND TRIANGLES.

1. Construct a rectangle (VI. 23), sides 5 inches and 4 inches. Divide it up into square inches using your set squares (VI. 25). What is its area in square inches?

2. Cut out a paper rectangle $ABCD$ (Fig. 138), sides 5 and 4 inches. Now through A rule any

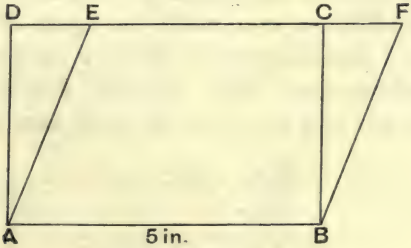


FIG. 138.

slanting line AE and cut off the triangle ADE and place it in the position BCF. Is BF parallel to AE? Is CF in a straight line with EC? If so, the figure $EABF$ is a parallelogram (VI. 20). What is its area?

3. Cut out a rectangle $ABCD$ of any convenient size. An ordinary sheet of paper will not be too big, and this will save cutting out. Measure its length

and breadth in centimetres and millimetres (*i.e.* centimetres and decimals of a centimetre) and so find its area in square centimetres. Now supposing *ABCD* (Fig. 139) to be the rectangle, draw a slanting line *BE*.

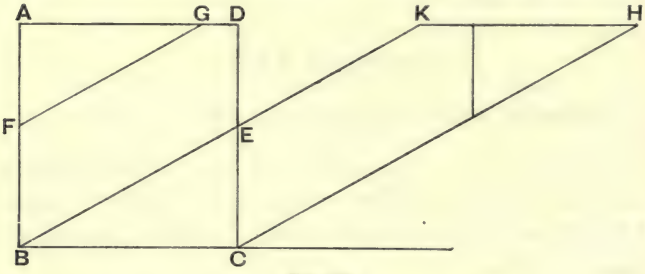


FIG. 139.

Mark off *BF* equal to *EC* and draw *FG* parallel to *BE*. Now cut out the pieces *AFG*, *FBEDG*, and fix them on to *BEC* so as to make the figure *BCHK*. Is this figure a parallelogram ? What is its area ?

4. Parallelograms like *ABCD*, *EBCF*, *GBCH* (Fig. 140) are said to be **on the same base** *BC* and to

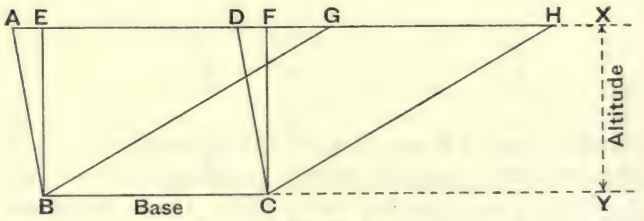


FIG. 140.

have the same height or **altitude** *XY*. The altitude is the perpendicular or shortest distance between the base and the opposite side. Which of the paral-

lelograms in Fig. 140 has the largest area? Measure if you can the area of any one of them in square centimetres.

5. Draw a figure similar to Fig 140 on squared paper and compare the areas by counting the squares. (Remember the instructions in VII. 4.)

6. All parallelograms on the same base and having the same altitude (or, as Euclid says, between the same parallels) are equal in area. What is the area of a parallelogram the base of which is 3 inches and height 2 inches?

7. Construct a parallelogram, sides 3 inches and 4 inches, one angle 60°, and find its area in two ways (Fig. 141.) If AB is taken as the base, then XY

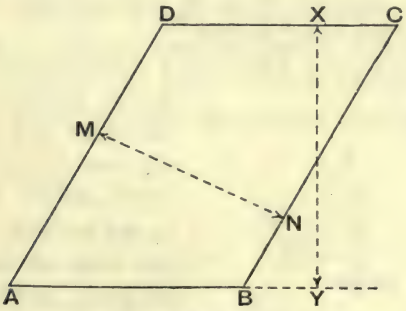


FIG. 141.

(use your set squares) is the altitude. Or, if AD be taken as base, then MN is the altitude. Do your two results agree?

8. Repeat the method of Ex. 7 with a parallelogram with sides 5 and 4 inches and contained angle 45°.

9. A parallelogram with all its sides equal is called a **rhombus,** unless the angles are also right angles, when there is a special name for the figure. (What is this name?)

Make a rhombus, sides each 4 inches, one angle 75°. Measure the other angles, and find the area of the figure.

10. Make a rhombus, side 2 inches, one diagonal 3 inches. Measure the other diagonal, the angles, and the area of the figure.

11. Construct any parallelogram carefully, drawing one diagonal, and cut it out. Now cut it carefully in two along the marked diagonal. The two triangular pieces should be exactly equal. Can you make them coincide? Replace them in position and note the position of the equal angles.

12. *ABCD* (Fig. 142) is any parallelogram and *AC* is one diagonal. Name the sides and angles of the two triangles *ADC*, *ABC*, which are equal to one another. Measure the area of the parallelogram, and hence find the area of one of the triangles.

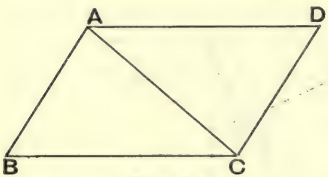


FIG. 142.

13. Every triangle can be regarded as half of a parallelogram. Construct a triangle with sides 2·7, 3, 3·3 inches, and by drawing parallels at two of its corners complete the parallelogram of which the triangle is half. Measure its area, and so find the area of the triangle.

14. It is not necessary to complete the parallelogram every time we want to find the area of a triangle.

In Fig. 143 the triangle ABC is half of the parallelogram $ABCD$, the area of which is base × altitude, *i.e.* $BC \times AE$. Now AE is the **altitude of the triangle**,

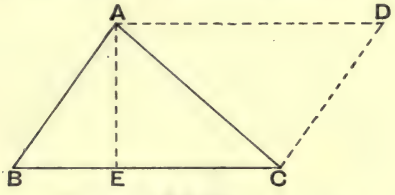


FIG. 143.

viz.: the perpendicular from A on the base BC. So the area of a triangle is half base × altitude. Measure these in Fig. 143, and find the area of ABC in sq. cms.

15. Sometimes the shape of the triangle is such that the perpendicular falls outside the base, as in

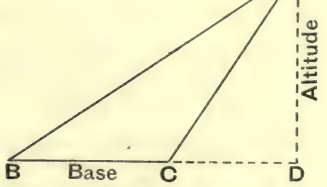


FIG. 144.

Fig. 144. Measure BC and AD and find the area of ABC in square inches. Then verify your result by drawing an exact copy of ABC on squared paper and counting the squares.

16. Any one of the three sides of a triangle can be regarded as the base, and for each base there is a different altitude. Thus, in Fig. 145, if AB be taken

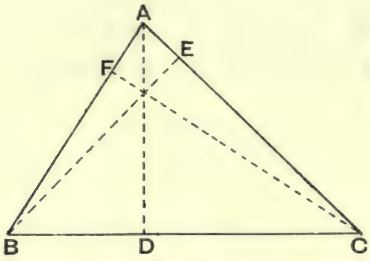


FIG. 145.

as base, the altitude is CF. Find the area of ABC in square centimetres by three different sets of measurements. Do your results agree?

17. Construct a triangle with sides 3, 3·5, 4 inches. Draw the three perpendiculars (using your set squares) and find the area in square inches in three different ways. Take the average of the three results by adding them together and dividing by 3. Then take the same measurements in centimetres and calculate the area in the same way, taking the average as before. Then find the number of square centimetres in a square inch. Thus:

$$\text{Area of triangle} = a \text{ sq. in.} = b \text{ sq. cm.}$$

$$\therefore 1 \text{ sq. in.} = \frac{b}{a} \text{ sq. cm.} \quad \text{(VII. 16.)}$$

18. Repeat the method of Ex. 17 with a triangle whose sides are 4, 6, 8 centimetres.

19. A triangle ABC has $AB = 3$ inches, $BC = 2$ inches, $\angle ABC = 60°$. Construct it and find its area.

20. In ABC, $AB=4$ in., $BC=3$ in., $\angle BAC=30°$. Construct and find its area.

21. $AB=4$ in., $\angle ABC=70°$, $\angle BAC=40°$. Construct and find area.

22. $AB=4$ in., $\angle ABC=60°$, $\angle ACB=30°$. Construct and find area.

23. Construct and find the area of an equilateral triangle, side 3 inches.

24. ABC is any triangle. Divide it into two equal *areas* by a straight line drawn through the point A.

25. In Fig. 146 the line AD is parallel to BC.

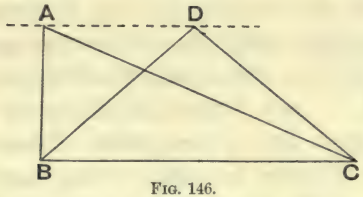


FIG. 146.

Which of the triangles ABC, DBC has the larger area? Give reasons for your answer.

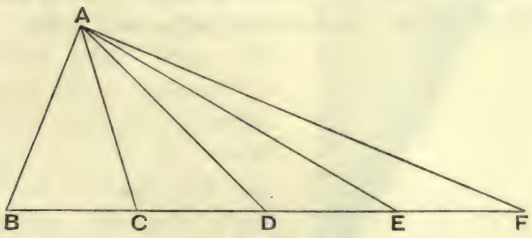


FIG. 147.

26. In Fig. 147 the line BF is divided into 4 equal parts. Which of the four triangles ABC, ACD, ADE, AEF has the largest area? Give reasons.

27. ABC is a triangle (Fig. 148). How many triangles can be drawn standing on the base BC and having the same area as ABC? There is one line

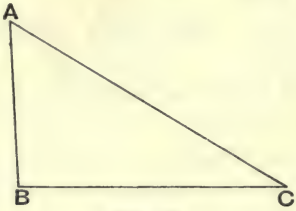


FIG. 148.

on which all the vertices (XV. 6) of these triangles lie: and this line may be called the **locus** of the vertices of all these triangles. Find this locus.

28. In X. 7 you found the locus of all points equidistant from two given points. Now show how to make an isosceles triangle on the base BC equal in area to ABC by finding where the two loci intersect.

29. Construct an isosceles triangle, base 2 inches, altitude 3 inches. Measure its angles and sides.

CHAPTER XIX.

CONSTRUCTION OF SQUARES AND OTHER QUADRILATERALS.

1. A **square** is a rectangle with all its sides equal. It can be constructed in the following way. Draw AB of the required length, say 3 inches. Then put your set square with one of the edges containing the right angle along the line AB, but projecting beyond A, as shown shaded in Fig. 149. Then put your straight edge along the longest side, hold it in position, and slide your set square up into the dotted position. Then you can rule the line AE which will be at right angles to AB. Cut off AC equal to AB. With centres at C and B and radius equal to AB describe arcs of circles meeting at D (Fig.

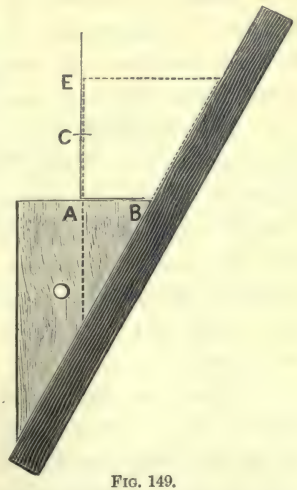


FIG. 149.

150). Join CD, DB. If your square is accurately drawn, the diagonal distances BC, AD should be equal.

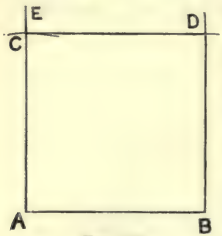


FIG. 150.

Measure them. Also the opposite sides of the square should be parallel. Test this with the set squares.

2. Make a square, side 2 inches, as in Ex. 1. Measure its diagonals.

3. Instead of describing two arcs meeting at D (Ex. 1), we might use set squares to draw through C a parallel to AB and through B a parallel to AC. Make a square of 6 cm. side in this way, and measure its diagonals.

4. Make a square of 4 inch side as in Ex. 3. Measure its diagonals. In future, either method may be used, and you are recommended to use whichever method you find more accurate.

5. The diagonals of a rectangle (VI. 23) should be equal in length. Make a rectangle, sides 5 inches and 3 inches. You can obtain a right angle as in Ex. 1 and the rest of the construction should be plain from Fig. 151. Measure its diagonals.

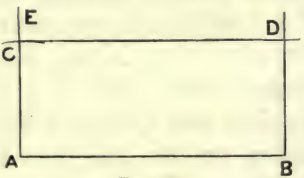


FIG. 151.

6. Make a rectangle, sides 7 cm., 4 cm., using a method similar to that in Ex. 3. Measure the diagonals.

7. Make a parallelogram, sides 4 inches, 3 inches, and contained angle 70°. It is important that you should be able to complete a parallelogram of which

two sides and the contained angle are given. Draw AB (Fig. 152) 4 inches long. Then with your protractor make the angle $BAE = 70°$. Cut off from AE with your pencil compasses $AC = 3$ inches. Now

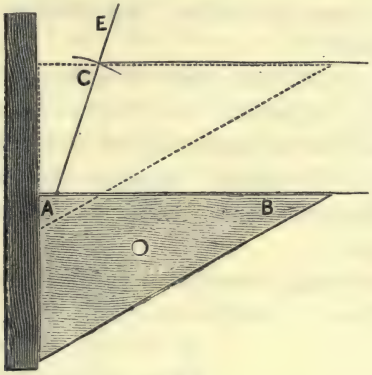


FIG. 152.

put your set square in the shaded position, and holding your straight edge in position slide the set square up till it passes through C. Rule your parallel. In the same way through B draw a parallel to AC. D,

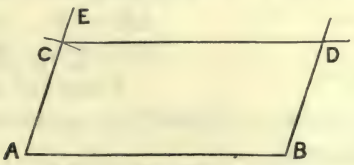


FIG. 153.

where these two lines meet, is the fourth corner of your parallelogram (Fig. 153). Remember that it is always better to draw the lines too long than too short.

8. A **rhombus** has all four sides equal. It differs from a **square** in not being rectangular. Construct a rhombus with side 2 inches and one of its diagonals 3 inches. Measure the other diagonal. Do the diagonals bisect each other. Are they at right angles to each other?

9. Make a rhombus, side 3 inches, one angle 60°. Measure the diagonals. Are they at right angles?

10. Construct a parallelogram, sides 4, 2·5 inches. One diagonal 4·5 inches. Measure the other diagonal. Do the diagonals bisect each other? Are they at right angles?

11. In naming a **rectilineal** figure (*rectus* = straight, *linea* = a line; ∴ rectilineal = what?) it is usual to go round in one direction. Thus, the quadrilateral in Fig. 154 may be called *ABCD*, or *ADCB*. Let Fig. 154 serve as a sketch to help you to draw a figure *ABCD* having the following measurements:

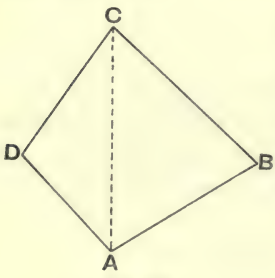


FIG. 154.

$AB = 3$ in., $BC = 3{\cdot}5$ in.,
$CD = 4$ in., $AD = 2{\cdot}5$ in.,
Diagonal $AC = 4$ in.

Why is it necessary to have five measurements for a four-sided figure?

12. $ABCD$ is a quadrilateral. $ABC = 90°$.
$AB = 2$ in. $BC = 2{\cdot}5$.
$CD = 3$ in. $DA = 1{\cdot}5$ in.

Make a rough sketch, and from it construct the figure. Find its area in square inches.

13. $ABCD$ is a quadrilateral. $AB=3$ inches.
$\angle ABC=100°$. $\angle BAD=110°$.
$\angle ABD=45°$. $\angle BAC=60°$.
Construct the figure and measure the lengths AD, BC, CD. Find the area of the figure.

14. C and D are two forts in the enemy's lines. We measure in our own lines a distance AB half a mile long which we call a **base-line.** We have a telescope which will revolve on a horizontal graduated circle. Fixing this at A, we look first at B, and then at C, and read carefully the angle that the telescope has

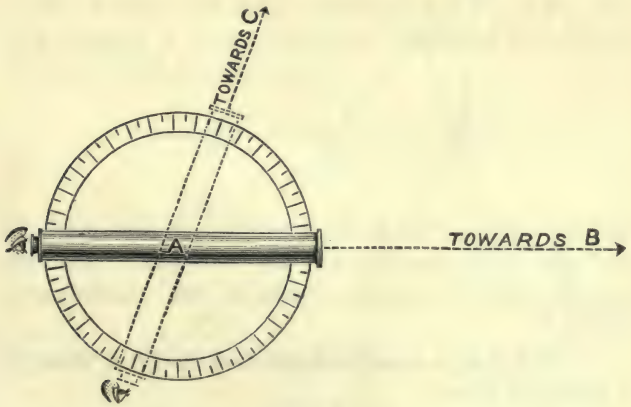


FIG. 155.

turned through (Fig. 155). This gives us $\angle BAC=72°$. Similarly we find the angle $BAD=103°$. Then we take our instrument to the other end B of our base line and measure the angles $ABC=85°$, $ABD=50°$. Now it is necessary to make a plan. Draw the plan,

taking 4 inches to represent 1 mile, and find how far the forts C and D are from the ends of the base line and from each other.

15. Sometimes it is necessary to make a triangle equal in area to a quadrilateral. In Fig. 156 $ABCD$ is any quadrilateral. BE is drawn parallel to the

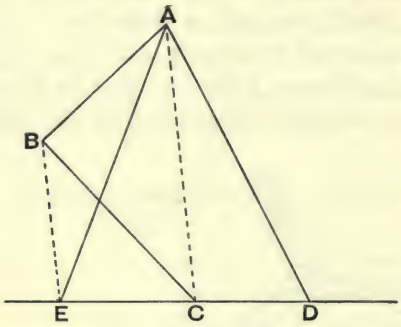


FIG. 156.

diagonal AC, meeting DC produced in the point E. Can you give reasons for the statement that the triangle AED is equal in area to the quadrilateral $ABCD$?

16. Make any quadrilateral and make a triangle of the same area.

17. Repeat this with the quadrilaterals whose measurements are given in Examples 12, 13. Find the areas of the triangles and compare your results with those previously obtained.

18. When you have found a triangle equal in area to your quadrilateral, you can bisect its base, and join the middle point to the vertex (Fig. 157). The line

AF bisects the triangle AED. (Why?) Does it also bisect the quadrilateral $ABCD$?

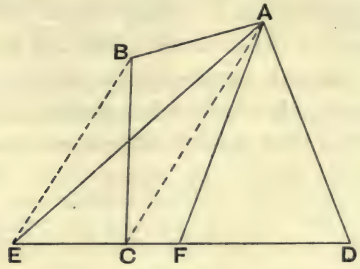


FIG. 157.

19. Answer the same questions for Fig. 158. Can you suggest a construction for bisecting the quadrilateral $ABCD$ in Fig. 158?

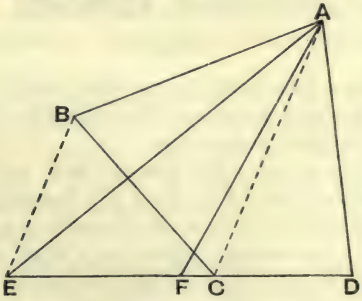


FIG. 158.

20. Make a quadrilateral $ABCD$ having

$AB = 5$ cm., $BC = 4$ cm., $\angle ABC = 90°$,

$CD = 3$ cm., diagonal $BD = 6$ cm.

Bisect it by a straight line passing through the point B.

21. Construct any quadrilateral of *irregular* shape. Bisect each of the sides. Join the middle points so as

to form another quadrilateral. What sort of quadrilateral is it? Can you give a reason? (X. 11.)

22. A **trapezium** is a quadrilateral which has *one* pair of opposite sides parallel (but not equal). Draw a pair of unequal parallel lines, and join the ends. Can the other pair of sides be parallel? Can they be equal? If the first pair of sides were equal as well as parallel, what would the figure be? Illustrate by drawings.

23. A **kite** is a quadrilateral having two pairs of adjacent sides equal. Construct a figure of this kind. Do the diagonals bisect each other? Are they at right angles?

ADDITIONAL EXERCISES.

XIX. a.

1. Describe a four-sided figure which shall have two sides parallel and one inch apart, and the other two sides equal, but not parallel, and two inches long. (K.)

2. Describe a square with side one inch, and on the four sides describe equilateral triangles; join the vertices, and show by measurement that the figure so formed is a square. (K.)

3. Draw a cross vertical section of a ditch whose depth is 3 feet, breadth at the bottom 2 feet, and at the top 4 feet, the sides being equal; and measure the sides. (K.)

4. Draw a square whose sides are 98 millimetres long. In it place another whose sides are 2·8 inches long and equidistant from the sides of the original square.

5. Construct a rectangle, sides 4·1 in. and 2·7 in. Measure the angles which a diagonal makes with two opposite sides.

6. $ABCD$ is a quadrilateral in which BC, AD are parallel. If $BC=15$, $CD=8$, $AD=7$, $BD=13$ inches, find the distance between AD and BC, and the length of AB.

7. Take any triangle ABC, and describe squares on the sides BA, CA, viz. $BFGA$ on BA, and $CKHA$ on CA; drop a perpendicular from A on BC, and join FC, KB: if the figure is drawn correctly, FC and KB will intersect on the perpendicular. (K.)

8. There is a piece of ground in the form of a trapezium, the lengths of the parallel sides of which are 20 and 34 yards, and the lengths of the other two sides 15 and 13 yards; find its area.

9. Draw the quadrilateral $ABCD$; $AB=3$ in., $BC=4{\cdot}5$ in., $CD=5$ in., $DA=2{\cdot}6$ in., and $ABC=110°$. Find its area in square inches.

10. Construct a square, side 3 in., measure the diagonals and circumscribe a circle.

11. Make a parallelogram with two sides 104 mm. and 65 mm., and included angle 60°. Measure the shorter diagonal, and the perpendicular distance between the longer sides.

12. Make a parallelogram with sides 84 mm. and 70 mm. and contained angle 58°. Draw the two diagonals and measure them.

13. Describe a rhombus, side 2·4 in., and included angle 75° (to be made by construction). In it inscribe a square.

14. Construct a triangle ABC with the sides 5 cm., 6 cm., 7 cm. respectively. Take P in AB so that $AP=2$ cm. and through P draw a line bisecting the triangle. Add the proof of your construction.

15. Construct the polygon $ABCDEF$, having $AB=4$ yards, $AF=3\frac{1}{2}$ yards, $AC=6\frac{1}{2}$ yards, $AD=8$ yards, $AE=6\frac{1}{2}$ yards, $BAC=35°$; $CAD=25°$; $DAE=40°$; $EAF=25°$. Find its area in square yards by reducing it to an equivalent triangle. Scale 2 yards to an inch.

16. The middle points of the alternate sides of a regular hexagon of side 4 inches are joined. Find the area of the equilateral triangle so formed.

17. On a base of $1\frac{1}{2}$ in. describe a regular pentagon. Reduce the figure to a triangle of equal area.

18. Make a parallelogram with two of its sides 5 in. and

3 in., and one angle 60°. Divide it into 5 equal parts by lines drawn from an angular point.

19. Draw the quadrilateral $ABCD$, having given $AB=3{\cdot}2$ in., $BC=2{\cdot}1$ in., $CD=3$ in., $DA=4{\cdot}1$ in., and $ABC=30°$ (to be made by construction). Find its area in square inches by reducing to an equivalent triangle.

20. $AB=1$; $BC=1{\cdot}2$; $CD=1{\cdot}4$; $DE=1{\cdot}6$; $EF=1{\cdot}8$; angle $ABC=140°$; $BCD=130°$; $CDE=120°$; $DEF=110°$. Construct a polygon $ABCDEF$ with these data, and measure the side FA; scale 1 inch $=0{\cdot}8$; and show that BC is parallel to FE. (K.)

21. What is a rhombus? How many measurements would you have to make to be able to reproduce it (1) in shape, (2) in both size and shape?

In each case mention a measurement or set of measurements that is just enough.

22. In a four-sided field $ABCD$, AB and AD are each 96 yards; $BC=64$ yards; $CD=48$ yds.; and BCD is a right angle. Find its area to the nearest sq. yd.

CHAPTER XX.

EUCLID I. 47.

1. Construct a square *ABCD* (Fig. 159) with side 2 inches. Draw the diagonal *BD* and construct a square on *BD*. If your work is accurate, the sides *BC*, *DC* produced will pass through the points *E*, *F*. Measure *CE*, *CF*. They should each be 2 inches long. Measure *BD*. What is its length? Divide *BD* by *BC* to two places of decimals.

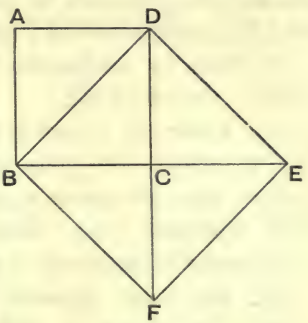


FIG. 159.

2. Repeat the above construction with *AB* 3 inches long. Then cut out the figure and cut it up into 5 right-angled triangles *ABD*, *BCD*, *DCE*, *BCF*, *ECF*. Are these triangles equal? How many of them does the square *ABCD* contain? How many the square *BDEF*? How many times is *BDEF* greater than *ABCD*?

3. The square on a line 1 inch long is 1 square inch. What is the size of the square on a line 2 inches long? 3 inches? 4 inches? 5 inches?

4. A square is 100 square inches in area. What is the length of its side?

5. A square is 49 square centimetres in area. What is its side?

6. A square is 2 square inches in area. What is the length of its side?

7. A square, side 1 inch, is drawn, and a square is described on one of its diagonals. What is the area of this square? (Ex. 2.)

8. Construct carefully a square of any size you please. Measure the diagonals. (They should be exactly equal.) Divide the length of the diagonal by the length of the side to two places of decimals. (Also find the square root of 2 to two places of decimals.)

9. Find by arithmetic the **square** of 1·41; also of 1·414; also of 1·4142.

10. Draw on squared paper a square whose side is 1·4 inches. (If your paper does not show tenth inches, take 10 spaces as representing 1 in. Then 14 will represent 1·4.) Now count the squares. The area ought to be nearly 2 square inches.

11. On your squared paper make a triangle ABC with AB 2 in., BC 2 in., $\angle ABC = 90°$. Make a square on each of the sides. What is the area of the big square? What is the area of the other two squares added together?

12. Make a triangle ABC with $\angle ABC$ 90° and $AB = 4$ in., $BC = 3$ in. What is the length of AC? What is the area of the square on AC? What is the area of each of the squares on AB, BC? Add these two areas together and compare with the square on AC.

13. Make a triangle ABC having $AB=4{\cdot}8$ in., $BC=1{\cdot}4$ in., $\angle ACB=90°$. Measure AC. Find the areas of the squares on the sides. Is the larger square equal to the other two added together?

14. Make a triangle with sides 12, 13, 5 cm. Measure its angles. Find the areas of the squares on the sides. Add the two smaller squares together.

15. In Fig. 160 a method of fitting two squares together to make one larger square is shown.

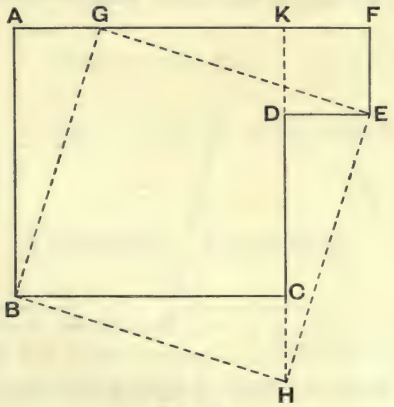


Fig. 160.

$ABCDEF$ is a figure made up of two squares, $ABCK$ and $KDEF$. Draw such a figure and cut it out in paper. Mark AG equal to DE or EF and join BG, GE. Now if the triangle ABG be cut off and put in the position DEH, and the triangle GEF cut off and put in the position BCH, the figure will be $GBHE$. Is this a square? Test this by measuring its sides and its angles. It is plain that $EH=BG$

(if it is plain, give a reason). Can you give a reason why GE is equal to BG?

16. Does the figure (160) tell you anything about the squares on the sides of the right-angled triangle ABG? Is the same thing true of all right-angled triangles?

17. It is usual in speaking of the angles of a triangle ABC to use the single letters A, B, C. Thus, in Fig. 161, A means the angle BAC of the triangle, B means the angle ABC, C means the angle ACB. When we are speaking of the lengths of the sides we

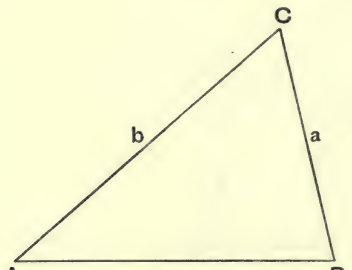


FIG. 161.

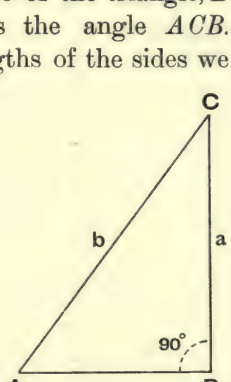


FIG. 162.

use small letters. Thus, a means the side opposite to the angle A, *i.e.* the side BC. Now if ABC (Fig. 162) be a right-angled triangle with the angle at B a right angle we find by measurement that $b^2 = a^2 + c^2$. This is a short way of expressing a fact about the sides of the right-angled triangle ABC. State this fact in words instead of symbols.

[This fact about right-angled triangles is said to have been discovered by Pythagoras (B.C. 500). It is proved in Euclid, Book I., Prop. 47.]

18. In Fig. 162, given $a = 6$ cm., $c = 8$ cm., $B = 90°$; find b. Do this both by construction and calculation.

19. In the same figure, given $a = 10$ cm, $B = 90°$, $b = 26$ cm.; find c.

20. The side opposite the right angle is called the **hypotenuse**. In a right-angled triangle, the sides containing the right angle are 15 cm., 8 cm. Find the length of the hypotenuse.

21. The hypotenuse is 3·3 inches, one side is 2·7 inches. Construct and find the length of the third side.

22. Make a square equal to the sum of two given squares.

23. Make a square equal to the difference of two given squares.

ADDITIONAL EXERCISES.

XX. a.

1. If the square described on one of the sides of a triangle be equal to the sum of the squares described on the other two sides of it, the angle contained by these two sides is a right angle.

Use this proposition to construct a right angle with your instruments.

2. Construct a square whose diameter shall be two inches.

3. Find the side of a square equal in area to the sum of two squares having sides of 20 feet and 30 feet, correct to an inch.

4. The radius of a circle is 13 inches and the length of a chord of it 10 inches; find the distance of the chord from the centre.

5. The radii of two intersecting circles are respectively 15 inches and 13 inches, and the common chord of the circles

24 inches long. What length of the line joining their centres lies within both circles?

6. Huntingdon is 60 miles due N. of London; Swindon is 70 miles due W. of London; Burton-on-Trent is 90 miles due N. of Swindon. Find the distance between Burton-on-Trent and Huntingdon, *either* by calculation from a rough sketch, *or* by measurement from a figure drawn to scale.

7. If a quadrilateral is drawn so that its diagonals are at right angles to each other, prove that the sum of the squares on two opposite sides is equal to the sum of the squares on the other two sides.

8. A and B are two fixed points, and a point P moves on a straight line at right angles to AB. Prove that the difference of the squares on AP, BP is constant.

9. In an acute-angled triangle ABC, AO is drawn perpendicular to BC. Denoting this perpendicular by p, and OC by x, equate the values of p^2 given by the two right-angled triangles and so determine x and thence p in terms of a, b, c (the lengths of the sides of the triangle). Deduce an expression for the area of a triangle in terms of the sides.

State, without proof, whether the expression applies for triangles generally.

CHAPTER XXI.

CIRCLES.

CHORDS: CIRCLES INTERSECTING AND TOUCHING.

1. Take two points A and B and draw the right-bisector of the line AB. Take a point on this right-bisector as centre and describe a circle passing through A. It should pass through B. Why? (X. 7.) How many circles can be drawn passing through A and B? Draw 6 of them.

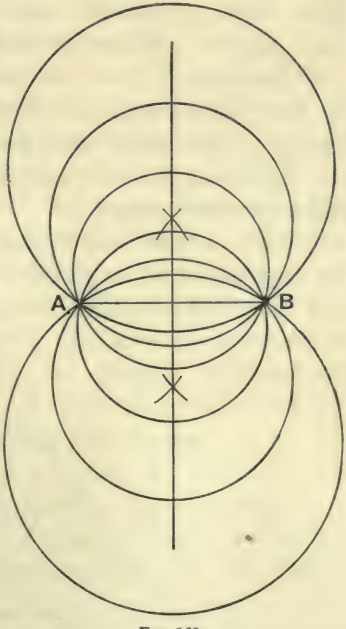


Fig. 163.

2. Can you draw a circle passing through A and B (Fig. 163) and having its centre off the right-bisector?

3. Take two points 2·3 inches apart. Draw the locus of the centres of all circles passing through these two points.

4. A straight line joining two points on a circle is called a **chord** of the circle. In Fig. 163 AB is a chord of every circle, and may be called the **common chord** of all the circles. Draw any two circles intersecting, and draw their common chord. Find its right-bisector.

5. Take three points A, B, C each 2 inches from the other two. Find the locus of the centres of all circles passing through A and B, and the locus of the centres of all circles passing through B and C. How many circles are there that pass through all three points? Where do their centres lie?

6. Describe a circle without using compasses, *e.g.* by drawing a pencil line round a penny. Draw two chords of this circle (not parallel chords) and draw the right-bisector of each chord. Where do these right-bisectors meet?

7. Give a clear description of a method of finding the centre of a given circle.

8. Fig. 164 shows an arc of a circle. How would

FIG. 164.

you find its centre, so as to be able to draw the rest of the circle?

9. Find the centres of the arcs in Fig. 165 and by measurements taken from your construction describe (i) a figure of exactly the same size and shape, (ii) a figure of the same shape but double the size. What do you mean by "double the size"?

FIG. 165.

10. Draw a circle on tracing paper. Draw a diameter, and fold the paper along this diameter. Do the two parts of the circle coincide? If they do, the circle is **symmetrical** about its diameter.

11. Draw any two circles on tracing paper, and draw the straight line joining their centres (sometimes called the **line of centres**). Fold the paper along this line. Do the two parts of the figure coincide? If so, the line of centres is a **line of symmetry** in the figure.

12. Draw two circles of radii 2 in., 1 in., with their centres 2·5 in. apart. In how many points do the circles meet? Join the points. Measure the distance between them. Is their common chord perpendicular to the line joining the centres?

13. Draw two circles of radii 2·7 cm., 3·6 cm., with their centres 6 cm. apart. Do they meet? In how many points? Join the points and measure the common chord? Is it perpendicular to the line of centres?

14. Draw two circles, radii 2·5 cm., 3·5 cm., with centres 6 cm. apart. In how many points do the

circles meet? Where they meet, draw a straight line perpendicular to the line of centres. Does this line cut either of the circles at any other point? Does the line of centres pass through the point where the circles meet?

15. Draw two circles, radii 2 in., ·7 in., with their centres 1·3 in. apart. Join the centres and produce this line. Does it pass through the meeting point of the circles?

16. Two circles whose centres are at a distance equal to the sum of the two radii meet in one point only and are said to **touch** at that point. Draw any two circles touching each other and draw the line of centres. Does it pass through the **point of contact**?

17. Is it possible for one circle to be inside another and touching it? If so, draw two such circles? Is the distance between their centres equal to the sum of the radii? If not, what is the connection between this distance and the lengths of the radii?

18. Draw two circles, radii 2 in., 3 in., touching one another **externally**. How far apart are their centres? Show the straight line joining them.

19. Draw two circles, radii 2 in., 3 in., the smaller circle touching the larger **internally**. How far apart are their centres? Show the straight line joining them and produce it. Does it pass through the point of contact?

20. Three circles of radii, 1 in., 1·5 in., 2 in. touch each other externally. How far apart are their centres? Construct the triangle formed by their centres and draw the circles.

21. Describe a circle of 2·5 in. radius, and two other circles, radii 1 in., 1·2 in., touching the large circle internally and each other externally. How far apart are their centres? Explain the method of construction, and in each case show the point of contact by joining the centres and producing (if necessary).

22. Construct a triangle ABC with sides 4, 5, 6 inches. With the points A, B, C as centres, describe three circles, each touching the other two externally. What is the radius of each circle? This can be found (i) by finding the incentre (XVI. 14.) of ABC, and drawing perpendiculars on the sides, (ii) by Arithmetic.

CHAPTER XXII.

THE CIRCLE. CHORDS AND TANGENTS. ANGLE IN A SEMICIRCLE.

1. Describe a circle of 2 inches radius. Mark off along a diameter of this circle distances of half an inch, and at each point draw a chord at right angles to the diameter. Measure each part of each chord. Are the chords bisected by the diameter?

2. Take distances of half an inch along a diameter of a circle of 2 inches radius, and at each point draw chords making an angle of 80° with the diameter. Are any of the chords bisected by the diameter?

3. In a circle, centre C, radius 2 inches, place a chord AB 3 inches long. Bisect AB at D and join CD. Measure the angles CDA, CDB, and the length CD.

4. Describe a circle of 6 cm. radius and draw a number of parallel chords at distances of 1 cm. Find their middle points and join them. What is the locus of their middle points? Can you find it by folding? Can you find it without folding?

5. In Fig. 166 suppose BC to be 8 cm. long in a circle rad. 5 cm. AD is drawn perpendicular to BC

from the centre A. Now D is the middle point of BC. (Why?) Therefore $BD = 4$ cm.

Also $AB^2 = BD^2 + AD^2$. (Why?)

Therefore 25 sq. cm. $= 16$ sq. cm. $+ AD^2$;

$\therefore AD^2 = 9$ sq. cm.; $\therefore AD = 3$ cm.

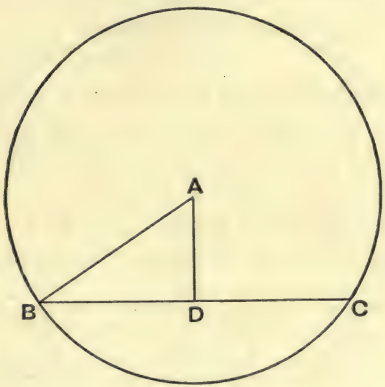


FIG. 166.

We have thus found AD by calculation. Now draw the figure carefully and measure the length of AD. Does it agree with our calculation?

6. A circle of radius 2·5 inches has a chord 3 inches long placed in it. Find by construction and by calculation the distance of this chord from the centre of the circle.

7. In a circle of 2·6 inches radius draw a diameter and mark off on it a point 1 inch from the centre. At this point draw a chord at right angles to the diameter. Find the length of this chord by measurement and by calculation.

8. A chord is drawn at a distance of 4 cm. from the centre of a circle of radius 5 cm. Find the length of the chord by construction and by calculation.

9. In Fig. 167 is shown a diameter with a number of cutting lines or **secants** at right angles to it.

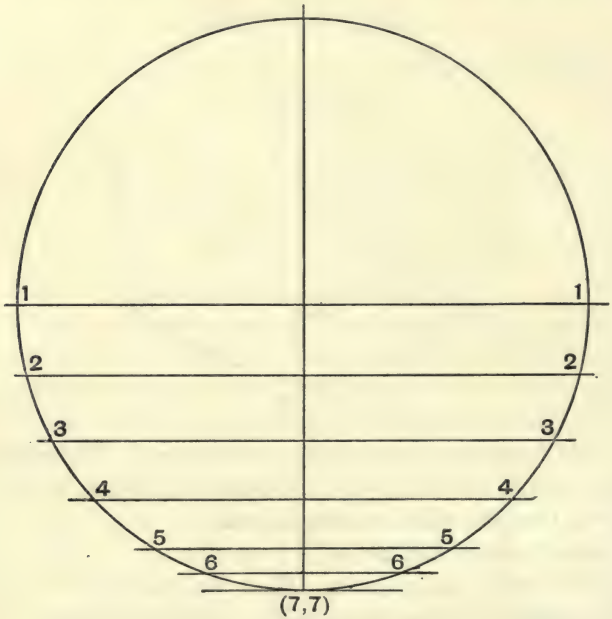


FIG. 167.

As the secant gets further from the centre, the distance between the **points of section** (11), (22), (33)... gets less and less until they coincide. The last line does not **cut** the circle in two points, but merely **touches** it in one point, for the two points (7, 7)

have come together into one. A line of this kind is called a **tangent** (touching line). Copy Fig. 167.

10. Draw a circle of 2 inches radius, and draw one of its radii. At the end of this radius draw a number of straight lines making different angles with the radius. All these straight lines except one will cut the circle in another point. Which straight line does not cut the circle?

11. Describe a circle of 2 inches radius, and draw two diameters AB, CD at right angles to each other. Draw carefully, using your set squares to obtain the right angle as shown in VI. 11, the 4 tangents to the circle at A, B, C, D. What sort of figure is formed by the 4 tangents? Measure its sides, angles, and diagonals.

12. Fig. 168 shows a semicircle, so CA, CD, and CB are all equal. The angle $CAD = 17°$.

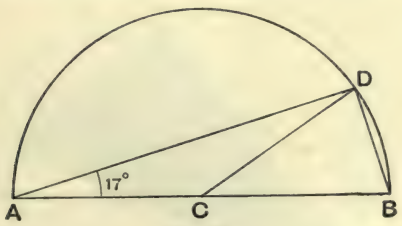


FIG. 168.

What is the angle CDA?
What is the angle ACD?
What is the angle DCB?
What are the two angles CDB, CBD?
What is the whole angle ADB?

13. If the angle at A were 20°, what would be the size of each of the other angles named in Ex. 12?

14. In Fig. 169 the point D is shown in many positions on the circumference of the semicircle.

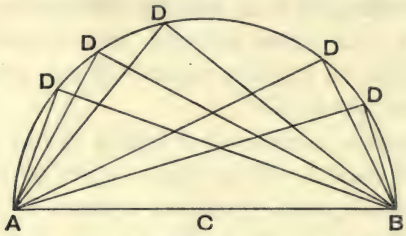


FIG. 169.

There is one angle of the triangle ADB which remains the same in all positions. Which angle is this, and what is the size of it?

15. Mark two points A and B on your paper and place your set square so that the two edges

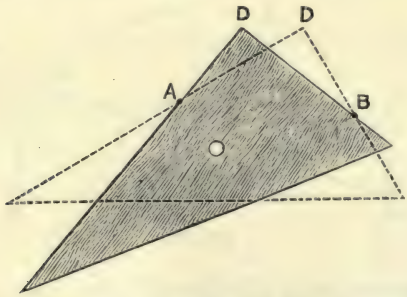


FIG. 170.

forming the right angles pass one through A and one through B. Mark on the paper the position of the right angle. You will find that there are

a great many positions for it. The shaded and the dotted positions in Fig. 170 show two of these positions. Mark a larger number of them and join them by freehand. What sort of curve do you get?

16. A and B are two fixed points 3 inches apart. P moves so that the angle APB is always 90°. What is the locus of the point P? If C be the middle point of AB, what is the length of CP? If PC be drawn between PA and PB to cut AB in C so that the angle CPA is equal to the angle CAP, show that the angle CPB must be equal to CBP. Hence prove that C must be the middle point of AB.

17. Euclid says in Book III. Prop. 31, "The angle in a semicircle is a right angle." What does Euclid mean by the angle in a semicircle?

18. Fig. 171 shows how to draw from a point A outside a circle a straight line AP **touching** the circle

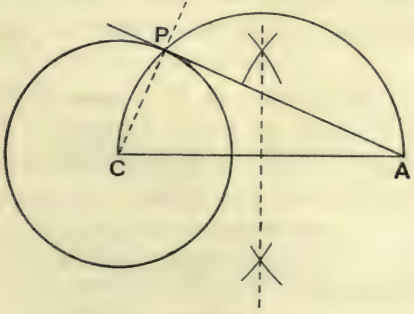


FIG. 171.

at P. C is the centre of the circle, and a semicircle is described on AC. Copy the construction, and explain why AP is a **tangent** to the circle.

19. Describe a circle of 1·5 inch radius and take a point 4 inches from the centre. From this point draw a tangent to the circle, using the construction of Ex. 18. Can you draw more than one tangent? If you can, do so. Measure the length of the tangent, *i.e.* the distance of the point from the point of contact (AP in Fig. 171).

20. Draw a circle, centre C, radius 2·5 inches, and take a point A 6·5 inches from C. From A draw two tangents AP, AQ to the circle. Measure their lengths. The chord PQ joining the points of contact is called the **chord of contact.** Measure the length of the chord of contact in this case.

21. Draw two **concentric** circles (circles having the same point as centre) of radii 5 cm. and 3 cm. Draw 6 straight lines which are to be chords of the outer circle and tangents to the inner circle. Measure the lengths of these chords. Are they equal? Can you obtain the length by calculation? (XX. 17.)

22. Describe a circle of 2·7 inches radius and in it place a chord 4 in. long. Describe a concentric circle passing through the middle point of this chord. Does the circle touch the chord? By means of this circle draw two other chords equal in length to the first, parallel to it and at right angles to it. Explain your construction.

23. Describe a circle, centre C, radius 2 in., and take a point A 5 in. from C. Through A draw a **secant** APQ, so that the chord PQ may be 2·5 inches long. (The secant should be a tangent to a concentric

circle obtained by drawing any chord 2·5 inches long and finding its distance from the centre.)

24. Fig. 172 shows one method of making a right-angled triangle of which the hypotenuse and one side

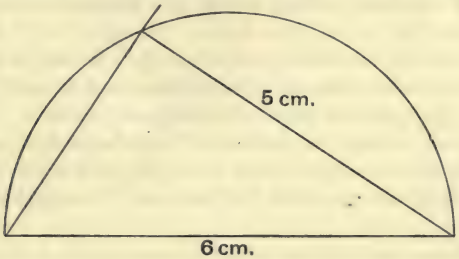


FIG. 172.

are given. Explain the method and employ it to find the third side of a right-angled triangle of the dimensions shown on the figure.

25. Employ the method of the preceding exercise to construct a rectangle, one side 3 in., diagonal 4 in. Measure the other side and diagonal.

ADDITIONAL EXERCISES.

XXII. a.

1. Draw a tangent to a circle two inches in radius from a point three inches from its centre.

2. By drawing tangents circumscribe an equilateral triangle to a circle of 1 in. radius.

3. Draw two different circles to touch the same given line at the same given point. Such circles are said to touch one another. Draw figures for both cases, (1) when they touch on the same side of the line, and (2) on opposite sides of it. (K.)

4. Describe a square of 3 inches side, inscribe a circle in the square, and form an octagon by cutting off the corners with symmetrically placed tangents to the circle. (K.)

5. Show by a figure that the tangents at the angular points of a triangle inscribed in a circle intersect the opposite sides produced in three points which are situated in a straight line. (K.)

6. Tangents are drawn to a circle (two inches in radius) at two points on the circumference 3 inches one from another ; find with the protractor at what angle they will meet. (K.)

7. A man stands on the top of a globe 15 feet in radius ; supposing his eye to be 5 feet from the surface of the globe, how far from his eye is the furthest part of the globe which he can see ? (K.)

8. Take any six points in the circumference of a circle, and draw tangents at them, thus forming a circumscribed hexagon : then join each pair of opposite angular points : the three lines joining the pairs of opposite angles will be found to pass through the same point. (K.)

9. Draw a circle, radius 4 cm. Take a point 9 cm. from the centre, and draw a line from it to cut the circle so that the chord cut off is 4 cm. in length.

Add the proof of your construction.

10. Take any two points C and P, $2\frac{3}{4}$ inches apart ; with centre C, radius $1\frac{1}{4}$ inches, describe a circle, and from P draw a straight line PQR, cutting the circle in Q and R, so that the length of the chord QR may be $1\frac{1}{2}$ inches.

11. Draw two tangents to a circle, radius 1·4 in., from a point 3·5 in. from its centre.

12. Draw two circles, radii 1·7 in. and 2·2 in., to touch each other externally. Draw tangents to each circle from the centre of the other.

13. Draw a tangent to a circle, radius 1·5 in., from a point 4 in. from the centre. Draw another tangent to the circle which shall be at right angles to the first.

14. Draw a straight line, and taking a point $1\frac{1}{2}$ inches distant from it as centre, describe a circle of 1 inch radius. Draw a

straight line touching this circle, and making an angle of 40° with the straight line.

15. To draw a tangent touching both of two given circles, each of which is exterior to the other.

Construction. Take any two circles, with unequal radii. Let A be the centre of the greater circle, B of the less. Round A describe a third circle, with radius equal to the difference of the two circles; from B draw a tangent to the new circle, touching it in C; join AC, and produce it to cut the outer circle in D; then at D draw a parallel to BC, touching the larger circle; it will be found also to touch the other. (K.)

16. Take another pair of circles, and repeat the construction of the preceding exercise, with this difference that the radius of the third circle (centre A) is to be equal to the *sum* of the radii, instead of the *difference*. Another tangent common to both circles will be found. (K.)

17. Draw two circles, radii 1 in. and 1·4 in., to touch one another, and draw the three common tangents.

18. Draw a circle with a radius of 8 centimetres (or if you prefer it, 3 inches,) and from a point P distant 16 centimetres (or 6 inches) from the centre O of the circle draw a tangent to the circle; measure and state its length.

If any secant PAB is drawn cutting the circle at A and B, show as well as you can by measurement and calculation, that the rectangle $PA \,.\, PB$ is equal to the square on the tangent from P to the circle. (Two or three secants should be drawn and measured.)

19. Take two circles, with equal radii, and draw a common tangent to them.

20. Describe two circles with radii 2 and 3 inches, the centres being five inches apart, and describe *three* common tangents to them.

21. Describe two circles with radii 1 and 2 inches, the centres being five inches apart, and describe four common tangents to them.

22. Draw two circles with radii 1·4 in. and 2·1 in., and their centres 4 in. apart. Construct all their common tangents.

23. Draw two circles centres A, B, radii 4 cm., 2 cm., distance AB 9 cm. Find the points S_1, S_2, on AB and AB produced which divide it in the ratio 2 : 1. (XXV. 19.) Draw a tangent from S_1 to one circle and produce. It should touch the other circle. The same is true for S_2. Measure the lengths of the tangents. Their ratio should be 2 : 1. Draw secants through S_1 or S_2. Measure their lengths. The ratio should again be 2 : 1.

24. The facts in the preceding exercise may be explained thus: Draw two squares of different sizes but with their sides parallel. The lines joining the corresponding vertices should meet in a point, corresponding to S_1 or S_2. Draw two regular hexagons or any similar figure with sides parallel. The lines joining corresponding vertices should again meet in a point. Circles may be regarded as similar polygons with the number of sides infinite. The points S_1 S_2 are called **centres of similitude.** [For this and the preceding exercise the author is indebted to Mr. J. S. Yeo of Fettes College.]

25. Draw the four common tangents to two circles of radii $\frac{5}{8}$ in. and $1\frac{1}{4}$ in., whose centres are $4\frac{1}{4}$ in. apart, and measure the distances of each centre of similitude from the centres of the circles. Find the ratio of these distances for each centre of similitude.

CHAPTER XXIII

CIRCLES SATISFYING THREE CONDITIONS.

1. AB (Fig. 173) is a straight line, and a circle of 1 inch radius is in contact with it. Now

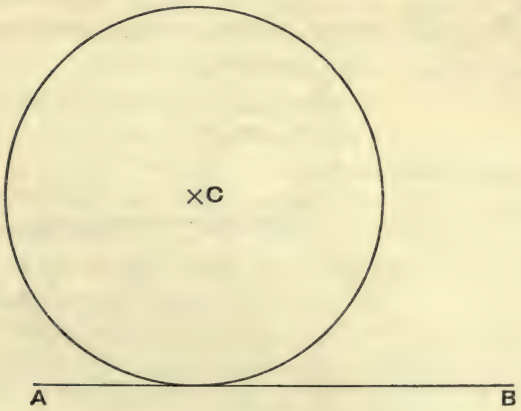


FIG. 178.

suppose the circle to roll along the line AB. What path will be taken by the centre of the circle C?

2. What is the locus of the centres of all the

circles of 1 cm. radius that touch the straight line XY? (Fig. 174.)

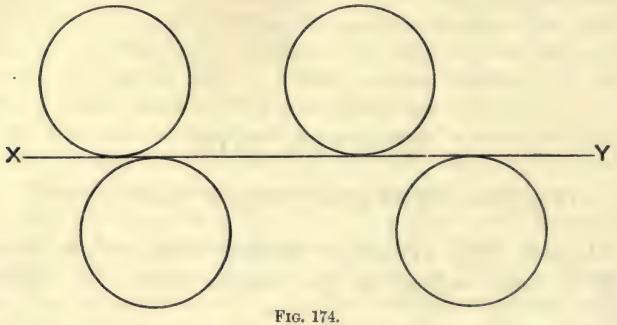


FIG. 174.

3. Fig. 175 shows several circles touching the

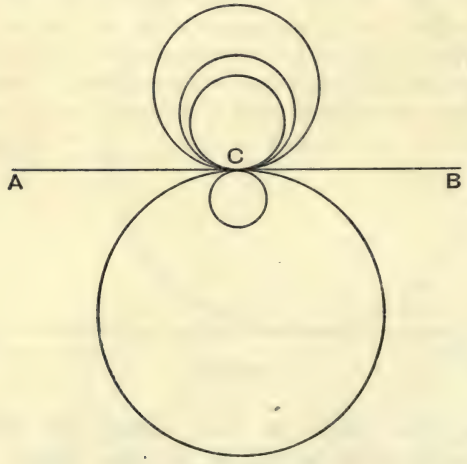


FIG. 175.

straight line AB at the point C. What is the locus of the centres of all the circles that touch AB at C?

4. Describe circles of 1·5 in. radius above and touching a given straight line at three points at distances of 1 inch along the line. Show how the points may be obtained by drawing (i) the locus of the centres of all circles of 1·5 in. radius touching the given line on the upper side; (ii) the loci of the centres of all circles touching the straight line at the given points.

5. Find and draw the locus of the centres of all circles of 1 inch radius touching a given circle of 2 inches radius (i) externally, (ii) internally. (XXI. 16, 17.)

6. Describe a circle of 2 inches radius. Draw a straight line at a distance of 1·5 inches from the centre of this circle. Then, by finding the point of intersection of two loci obtained as in Ex. 2 and 5, describe a circle of 1 inch radius touching the straight line and touching the first circle externally.

How many such circles are there?

Is it possible to make the second circle touch the first internally instead of externally? If so, indicate the construction.

7. What is the locus of the centres of all circles passing through 2 given points?

8. Take two points A and B 2 inches apart and describe a circle of 1·5 in. radius passing through A and B. The centre is to be the intersection of two loci, and can be obtained in two different ways.

9. What is the locus of the centres of all circles touching two given straight lines? (XII. 12.)

10. Draw two straight lines ABC, DBE crossing at an angle of 70°, and describe a circle of 1 inch radius touching both lines. How many such circles

can be drawn? Show that the problem can be solved (i) by combining Ex. 2 and 9; (ii) by Ex. 2 alone.

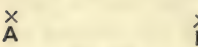


Fig. 176.

11. Show how to describe a circle (i) having a given centre A and passing through a given point B.

(ii) Having a given centre A and touching a given straight line BC (Fig. 177).

Fig. 177.

(iii) Having a given centre A and touching a given circle (Fig. 178).

Fig. 178.

12. Can a circle be drawn with centre A passing through both B and C (Fig. 179)? Why not?

Fig. 179.

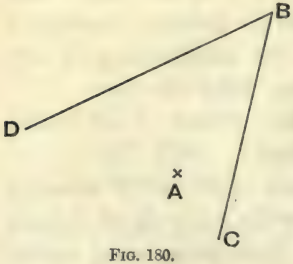


Fig. 180.

13. Can a circle be drawn with centre A touching both BC and BD (Fig. 180)? Why not?

14. Can a circle be drawn with centre A and touching both the given circles (Fig. 181)? Why not?

15. If the centre of a circle is given, how many more conditions may the circle be made to satisfy?

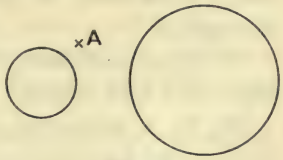


FIG. 181.

16. A circle can be made to satisfy 3 conditions, *i.e.* to do 3 things such as the following:

(1) pass through a fixed point,
(2) touch a fixed straight line,
(3) touch a fixed circle,
(4) have a radius of a fixed length;

(but giving a circle a fixed centre is equivalent to making it do two things). You have found how to make a circle pass through 3 fixed points and touch 3 fixed straight lines: also how to make a circle of fixed radius pass through 2 fixed points or touch 2 fixed straight lines. Now invent a construction for a circle of 1 inch radius passing through A and touching BC (Fig. 182). (Copy this figure on your paper and make the construction there.)

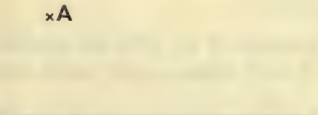


FIG. 182.

17. Construct a circle of 3 cm. radius passing through A and touching the circle (Fig. 183).

FIG. 183. A

18. Construct a circle touching BC at D (to how many conditions is this equivalent?), and passing through A.

19. In a semicircle of 2 inches radius inscribe a circle of 1 inch radius.

20. In a semicircle of 2 inches radius inscribe a circle of 0·7 inch radius.

21. Are there any other combinations of three of the conditions mentioned in Ex. 16 that you can devise for yourself? You will find some that will give you considerable trouble in solving, *e.g.* "Describe a circle touching 3 given circles" is a very difficult problem; but there are others which are easier, and some which you will meet with later on.

ADDITIONAL EXERCISES.

XXIII. a.

1. Draw two lines making an angle of 45° with one another, and describe all the circles of $\frac{1}{2}$ inch radius which touch both lines.

2. Draw a circle of radius 1 in. with centre 1·6 in. from a straight line AB. Draw a second circle of radius $1\frac{3}{4}$ in. which shall touch AB and be touched internally by the first circle.

3. Describe a circle of 1 in. radius, touching two straight lines inclined at an angle 45°: and describe two more circles also touching the same two lines and the next circle.

4. Draw two circles, radii 1 in. and 1·5 in., to touch one another externally. Draw a third circle, radius 4 in., to touch and enclose them both.

5. Draw two circles, radii 0·9 in. and 1·6 in., to touch each other externally, and a third, radius 2·6 in., to touch and enclose the other two.

6. Draw two straight lines inclined at an angle of 60° (to be made by construction). Describe a circle, radius 1·5 in., to touch the two lines.

7. Given a circle of 1·5 in. radius, take a point A, 3 in. from its centre, and a point B in the circumference 2 in. from A. Describe a circle which will pass through the point A, and touch the given circle at B.

8. The centres of three circles which touch are respectively 3, 2, $1\frac{1}{2}$ inches apart. Draw the circles. (XXI. 22.)

9. ABC is an isosceles triangle, right-angled at C, having $AC=3$ in. With centre A and radius $1\frac{1}{2}$ in. a circle is described. Required another circle touching this circle, and touching BC at B. Give two solutions if possible.

10. Draw a line AB, 3 in. long; describe a circle radius 0·75 in. to touch AB at A, and a second circle to touch AB at B and the first circle.

CHAPTER XXIV.

ANGLES IN A SEGMENT. EUCLID III. 21, 31.

1. Draw a circle ABC of any radius, and a diameter AOB through O the centre. Draw any other chord AC through A. Join OC. Measure the angle CAB, also the angle COB. Is there any connection between the numbers of degrees? Can you give a reason for such a connection? (XIII. 17.)

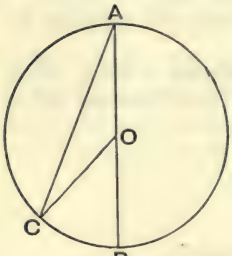


Fig. 184.

2. Copy Fig. 185 and measure the angles CAB, COB. Can you give any reason why CAB is half of COB? Do the same for Figure 186.

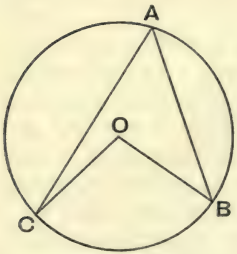


Fig. 185.

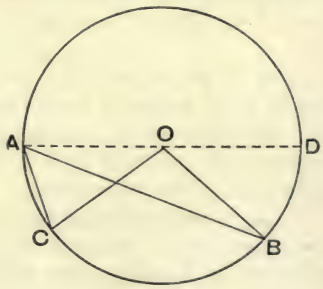


Fig. 186.

3. In Fig. 187 measure *CAB* and the **reflex angle** *COB*. How can you use your protractor to measure a reflex angle? (V. 4.) Is *CAB* half of the reflex angle?

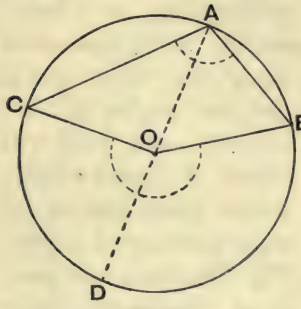


FIG. 187.

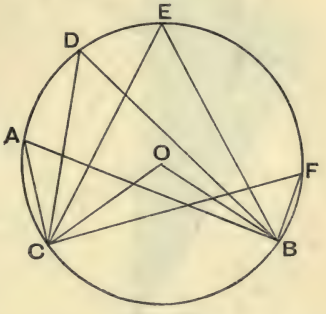


FIG. 188.

4. Which of the angles *CAB, CDB, CEB, CFB* is half of *COB*? Are they all equal? Test with your protractor. (Fig. 188.)

5. Are the angles *CAB, CDB, CEB* equal (Fig. 189)?

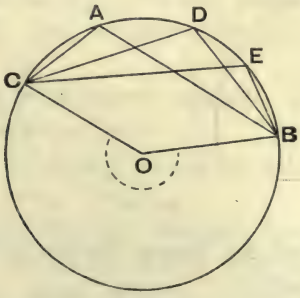


FIG. 189.

Is each half of the reflex angle *COB*? Test by measuring.

6. Look back to XXII. 15, where by using the right angle of your set square you found a number of points lying on a semicircle. Now, instead of using the right angle, use the angles of 60°, 30°, 45°, and you will obtain 3 curves of different sizes, but all of them portions of circles. Fig. 190 shows two positions of the set-square for an angle of 60°.

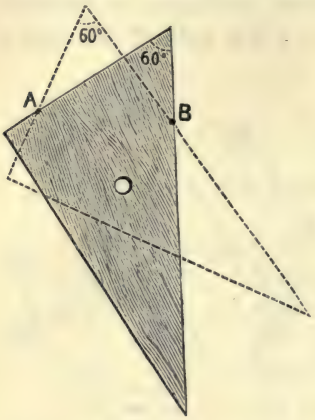


FIG. 190.

7. A figure made by a chord of a circle and the part of the circumference which it cuts off is called a **segment of a circle**. A semicircle is one form of

FIG. 191.

segment, but all the figures shown in Figure 191 are segments of circles.

Fig. 192 shows the arc of a segment without its chord. It also shows the way to obtain what Euclid calls **the angle in a segment.** You take any point

P on the **arc** and join P to the ends of the arc. Then APB is the angle in the segment. All the

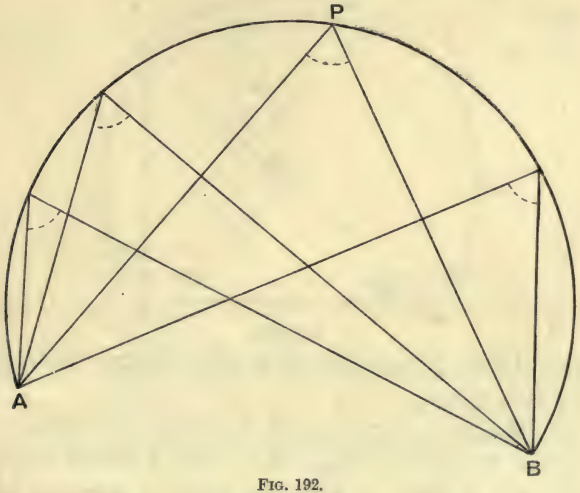


FIG. 192.

marked angles in Fig. 192 are **angles in the same segment**. Are they equal? Measure with your protractor.

8. Construct 4 different segments of circles and measure the angles in them.

9. Can you give any reason for all angles in the same segment being equal?

10. AB (Fig. 193) is a straight line. Suppose it is required to make on AB a segment of a circle containing an angle of 50°. Make $\angle ABC = 90°$, $\angle BAC = 40°$. What is $\angle ACB$? (Give a reason for your answer.) Now draw a circle passing through A, B, C. (XVI. 19.) Can you give a reason why the

centre should come at the middle point of AC? (See Chap. XXII.) If you can, it will be sufficient to

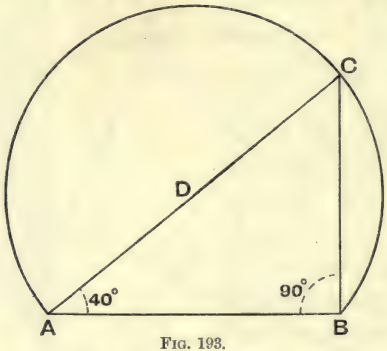


FIG. 193.

bisect AC at D and take D as your centre, when you copy this figure.

11. Make a segment of a circle containing an angle of 70° and standing on a chord 2 inches long.

12. On a base of 3 inches make a segment of a circle containing an angle of 80°.

13. On a base of 3 inches make a segment containing an angle of 90°.

14. If one angle of a triangle be 90° and another 80°, the third will be $90° - 80° = 10°$. Now suppose we have to make a segment containing an angle of 130°. If we use the construction in Ex. 11, the angle at B will be 90°, and the angle at A $90° - 130° = -40°$. What is the distinction between $-40°$ and 40°? An angle of $-40°$ must be made on the side of the line opposite to that on which the angle of 40° lies. So the construction for making a segment containing an angle of 130° is shown in Fig. 194. Notice that the angle of

40° is made below the line. Find the reason why the

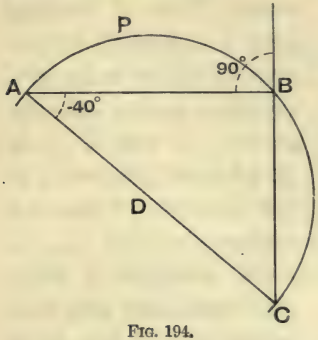


FIG. 194.

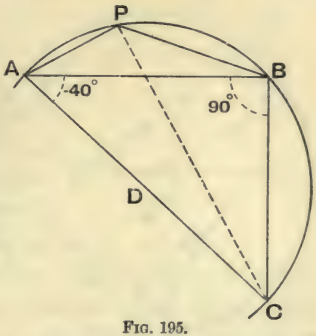


FIG. 195.

angle APB in Fig. 195 is 130°. What is $\angle APC$? What is $\angle CPB$? Copy Fig. 194.

15. On bases each 2 inches long construct segments of circles containing angles of 110°, 135°, 150°.

16. In Fig. 196 two of the angles are marked in

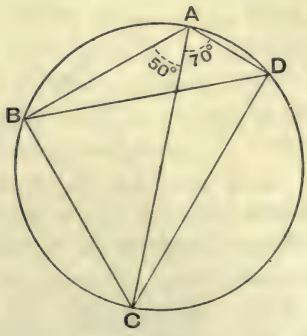


FIG. 196.

degrees. Find the number of degrees in the angles BDC, DBC, BCD.

What is the total number of degrees in $BAD+BCD$?

17. Fig. 197 shows a quadrilateral inscribed in a circle. Two of the corners are joined to the centre. What is the sum of the angles at the centre, A and B? If you know this, can you tell the sum of the two angles a and b?

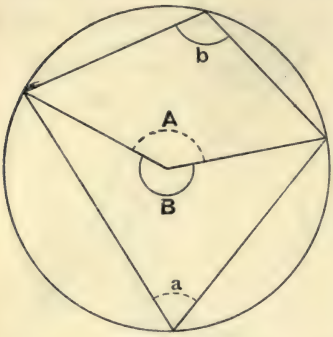


FIG. 197.

18. Describe a large circle and take any four points on it. Join them so as to form a quadrilateral. Measure a pair of opposite angles and add them together. Do this for 4 different quadrilaterals whose corners lie on a circle. (A quadrilateral whose corners lie on a circle is called a **cyclic quadrilateral**.)

19. What have you now found out about the opposite angles of a cyclic quadrilateral?

20. The dotted segment in Fig. 198 contains an angle of 50°. What is the angle in the other segment?

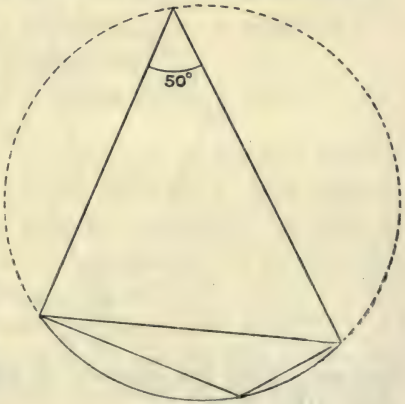


FIG. 198.

21. A point P moves so that AB always **subtends**

an angle of 70° at P. (Fig. 199.) What is the locus of P?

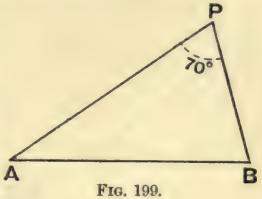


FIG. 199.

22. Construct a triangle on a base of 3 inches, having an altitude of 1 inch and a vertical angle of 70°.

23. On a base of 2 inches construct a triangle of area 1 square inch, having a vertical angle of 40°.

24. $ABCD$ is a quadrilateral inscribed in a circle. AC is a diameter, and $BAC = 45°$ and $DAC = 35°$. Find all the angles of the quadrilateral, and show that two of the sides are equal.

25. Describe a circle and draw AB touching it at C. Draw CD, a diameter, passing through the centre O. Draw CE to meet the circle in E, making the angle ECB 50°. Join ED. What is the size of the angle ECD? What is the angle CDE? Why are EDC and ECB equal? Is the angle ECA equal to the angle in the segment EFC? Repeat this construction with a different circle and angle, so as to verify the following statement:

If a chord be drawn from the point of contact of a tangent, the angles which it makes with the tangent are equal to the angles in the alternate segments of the circle.

26. Apply the preceding proposition to the problem given in Ex. 10, "To describe on a given base a segment to contain an angle equal to a given angle."

27. Using the method of the preceding exercise, describe a triangle having a base of 2 inches, an altitude $1\frac{1}{2}$ inches, and a vertical angle of 53°.

CHAPTER XXV.

RATIO. PROPORTION. SIMILAR TRIANGLES.

1. Take a straight line 3 in. long and divide it in the ratio of 2 to 5. Fig. 200 shows the construction. Look back to Chap. X. for the explanation.

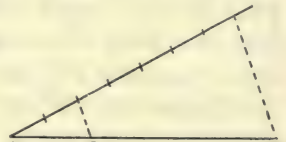


FIG. 200.

2. ABC (Fig. 201) is any triangle and DE is drawn parallel to BC. Measure in centimetres the lengths of AD and DB and divide the first by the second. Do not take the division beyond two places of decimals. (Why?) Then measure AE and EC, and divide AE by EC in the same way.

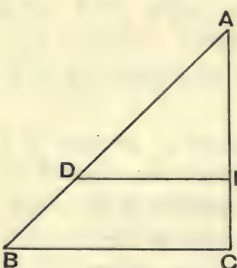


FIG. 201.

3. Draw a triangle for yourself and draw a straight line parallel to the base as in Fig. 201. Measure and divide as before. Is $\frac{AD}{DB} = \frac{AE}{EC}$? Repeat this with three other triangles and parallels.

4. In Fig. 202, find the value to 2 places of decimals of the fractions $\frac{FG}{FK}$, $\frac{FH}{FL}$, $\frac{GH}{KL}$. Do they agree?

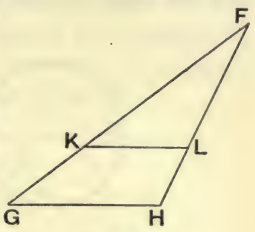


FIG. 202.

5. Make a square of side 3·4 inches and measure its diagonal. Divide the length of the diagonal by the length of the side. Compare your result with what you obtained in XX. 8.

6. If your measurements and constructions were sufficiently accurate, you would find that, for a square, diagonal ÷ side = 1·4142... correct to 4 places of decimals. This is sometimes expressed by saying that the **ratio** of the diagonal of a square to its side is 1·4142.... Find, to 2 places of decimals, the ratio of the altitude of an equilateral triangle to its base. Take the side of the equilateral triangle as 10 cm. Then take another equilateral triangle of side 3 inches and find the ratio of the two corresponding lines.

7. In Chap. IV. the ratio of the circumference of a circle to its diameter was found by measurement. The measurement of the circumference is a difficult one to make accurately, and if you found the ratio to be 3·14, you may be satisfied with your accuracy. The ratio correct to 5 places is 3·14159; but for practical purposes 3·14 and $3\frac{1}{7}$ are near enough. Which of these two is nearer to the truth?

8. Fig. 203 shows two circles of different sizes. But the ratio of the circumference to the diameter is

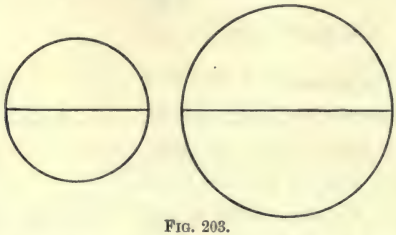


FIG. 203.

3·14159... for both circles. The same is true for any pair of circles. So we have this fact:

$$\frac{\text{1st circumference}}{\text{1st diameter}} = \frac{\text{2nd circumference}}{\text{2nd diameter}}$$

and the four are said to be **in proportion** or **proportionals.**

Definition.—Four quantities are said to be in proportion when the ratio of the 1st to the 2nd is equal to the ratio of the 3rd to the 4th. Thus if $\frac{a}{b} = \frac{c}{d}$, a, b, c, d are proportionals. The ratio $\frac{a}{b}$ is sometimes written $a : b$.

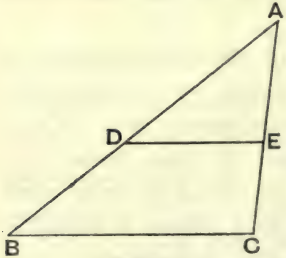


FIG. 204.

Now in Fig. 204 DE is parallel to BC. Find four lines in the figure which are in proportion, and write them down in correct order. Also find the value, to one place of decimals, of the equal ratios.

9. Draw two straight lines AB, DE, 3 inches and 4 inches long respectively. At A and D make angles of 70° and at B and E make angles of 50°, completing the triangles ABC, DEF as shown in the sketch (Fig. 205). Measure the angles at C and F.

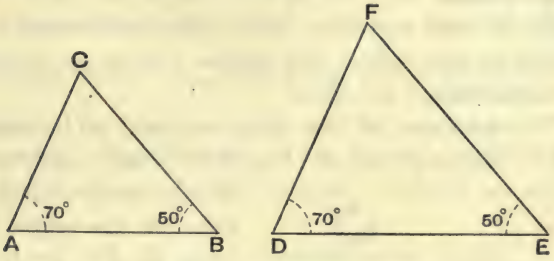


Fig. 205.

Now find the ratios $AC : DF$ and $CB : FE$. Also, find whether

$$\frac{AC}{AB} = \frac{DF}{DE},$$

$$\frac{AB}{BC} = \frac{DE}{EF},$$

$$\frac{BC}{CA} = \frac{EF}{FD}.$$

In each case take the calculation no further than two places of decimals. The measurements should be taken to the nearest hundredth of an inch.

10. Make a triangle with sides 4, 5, 6 cm., and another triangle with sides *half as much again*. Measure the angles of each triangle.

11. Using the methods of Ex. 9 and 10, but with different values, verify the following statements:

(i) If two triangles have the angles of the one respectively equal to the angles of the other, the sides about the equal angles taken in order are proportionals.

(ii) If two triangles have their sides (taken in order) in proportion, the angles (taken in pairs in the same order) are equal.

Two triangles of this kind are said to be **similar.**

12. The angle at A (Fig. 206) is 50°. Any point B is taken in one of the arms of the angle, and from B, BC is drawn perpendicular to the other arm. Five different positions for B and C are shown; but all the triangles are similar. Write down the number of degrees in the angles at B and C.

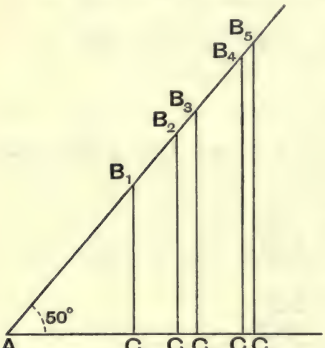


FIG. 206.

Since the triangles in Fig. 206 are similar, it follows that

$$\frac{B_1C_1}{AB_1}=\frac{B_2C_2}{AB_2}=\frac{B_3C_3}{AB_3}=\ldots,$$

and each of these ratios is found to be 0·766.... This ratio is called the **sine** of an angle of 50°, and is written **sin 50°**. Construct carefully an angle of 60°, make a right-angled triangle like *one of those in*

Fig. 206, and, by measuring the perpendicular and the hypotenuse, calculate the value of sin 60° (to 2 places of decimals). Similarly, find sin 70°, sin 40°, sin 30°. Remember that, if your angle is correctly made, the triangle had better be a large one.

13. Referring again to Fig. 206, you will see that every **perpendicular** B_1C_1, B_2C_2... has a corresponding **hypotenuse** AB_1, AB_2..., and the ratio $\frac{\text{perpendicular}}{\text{hypotenuse}}$ is always the same so long as the angle at A is unchanged. The third side of the triangle AC_1, AC_2... is called the **base**. It will be found that for an angle of 50°,

$$\frac{\text{base}}{\text{hypotenuse}} = 0{\cdot}6428..., \quad \frac{\text{perpendicular}}{\text{base}} = 1{\cdot}1918....$$

These ratios are called respectively the **cosine** and **tangent** of an angle of 50°, and are written **cos 50°**, **tan 50°**.

From the figures which you have drawn to find the sine of 60°, 70°, 40°, 30° you should be able to find the values of the cosines and tangents of the same angles correct to 2 places of decimals. Find them.

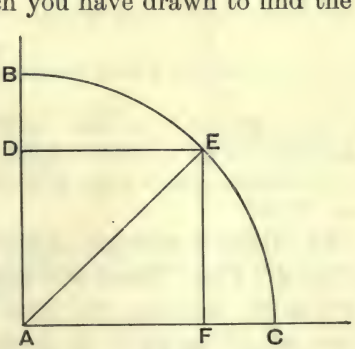


Fig. 207.

14. Fig. 207 shows the construction for an angle whose sine is 0·7. Make a **quadrant** of a circle (radius 10 cm.), drawing two radii AB, AC at

right angles. On AB mark off $AD=7$ cm. and draw DE parallel to AC to meet the circle at E. It is plain from the figure that

$$\sin EAC=\frac{EF}{EA}=\frac{AD}{AB}=\frac{7}{10}=0{\cdot}7.$$

By a similar construction find the angles whose sines are 0·5, 0·83.

15. If the base AF is marked off along AC, and FE drawn parallel to AB (Fig. 207), we can, in much the same way, construct an angle having a given cosine. Construct the angles whose cosines are 0·5, 0·83.

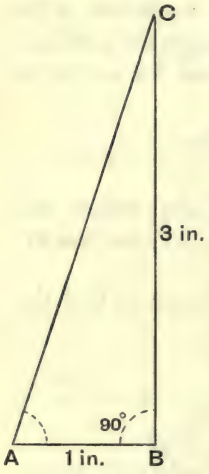


Fig. 208.

16. Fig. 208 shows the construction for an angle whose tangent is 3. AB is drawn 1 inch long. At B, BC is drawn perpendicular to AB and 3 inches in length. CA is joined. Then

$$\tan BAC=\frac{3 \text{ in.}}{1 \text{ in.}}=3.$$

By a similar construction find the angle whose tangent is 1. Also the angles whose tangents are 2, 3·7.

17. Draw a triangle ABC, having AB 6 in., BC 3 in., AC 5 in. Bisect the angle at B by BD cutting AC in D. Measure the lines AD, DC. What is the ratio of AD to DC? Is it the same as the ratio of AB to BC? Through C draw CE parallel to BD meeting AB produced at E. Measure BE. Mark 4

equal angles in your figure, and give reasons for their being equal.

18. Draw 3 triangles of unequal sides; in each of them bisect the vertical angle, and prove by measurement that the bisector divides the base into parts which are in the same ratio as the sides of the triangle.

19. Make a triangle as in Ex. 17. Produce AB to E and bisect the exterior angle CBE by BF cutting AC produced at F. What is the ratio of AF to CF? The line AC is said to be divided externally at F in the ratio of $AB : BC$.

ADDITIONAL EXERCISES.

XXV. a.

1. Construct an angle whose sine is 0·72; determine the angle and its tangent by measurement, and compare these results with those obtained from the tables. Take 10 cm. as the hypotenuse of the right-angled triangle you first construct. (*Army Entrance, July*, 1902.)

2. Construct the angle whose cosine is $\frac{2}{7}$, and find the values of the other ratios.

3. The tangent of one acute angle is 7, and the sine of another is 0·7; find graphically the cosine of the difference between the angles, explaining the constructions and measurements which you make.

Check your result by measuring the difference of the angles with your protractor and finding its cosine from the tables. (*London Matric. Sept.*, 1902.)

CHAPTER XXVI.

PROPORTIONALS.

1. If $\frac{a}{b}=\frac{c}{d}$ (sometimes written $a:b=c:d$), the four quantities a, b, c, d are proportional, and d is called a **fourth proportional** to a, b, and c. Take a numerical instance, $\frac{3}{4}=\frac{9}{12}$. 12 is a fourth proportional to 3, 4, 9. Notice the following facts, which are important, about these four numbers, for which $3:4=9:12$.

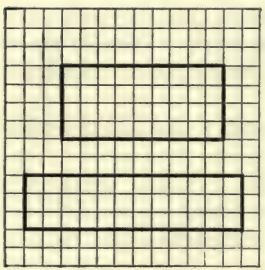


FIG. 209.

(i) $3\times12=4\times9$. This is shown on squared paper.

Write down the corresponding statement when

$$a:b=c:d.$$

(ii) $\frac{3}{9}=\frac{4}{12}$.

Write down the corresponding statement when

$$a:b=c:d.$$

2. Find, by algebra or by mental arithmetic, a fourth proportional to the numbers 1, 2, 3; also to the numbers 2, 3, 4; also to the numbers 6, 5, 3.

3. Fig. 210 shows a geometrical construction for finding a 4th proportional to 3 given lengths. Make

any angle at A. Mark off

AB = 1st length.

BC = 2nd length.

AD = 3rd length.

Join BD, and draw, through C, CE parallel to BD.

Then $AB : BC = AD$: some line in the figure. Which?

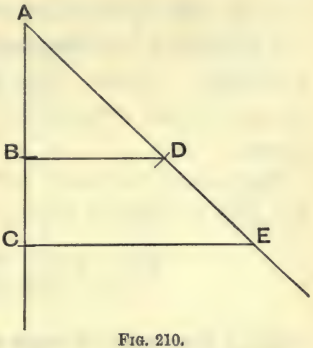


FIG. 210.

By this construction find a 4th proportional to 1·7 in., 1·1 in., 2·1 in. Measure it and verify your result by calculation. Your result should be correct to the nearest tenth of an inch.

4. Sometimes, to save space, the following method is used. You will understand it if you look back to XXV. 4.

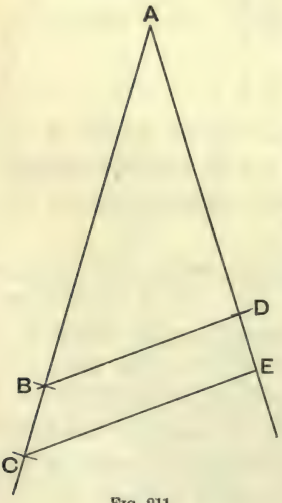


FIG. 211.

To find a 4th proportional to 5, 6, 4. Make $AB = 5$, $AC = 6$, $AD = 4$. (Fig. 211.) Join BD and draw CE parallel to BD. Then the 4th proportional is not DE, but the line corresponding to AC, viz. AE.

Use this method to find 4th proportionals to

4, 5, 6,

6, 5, 4,

5, 6, 4.

5. In the proportion $a : b = c : d$, the quantities a, d are called the **extremes**; b, c are the **means** (mean = middle). Notice (Fig. 209) that the rectangle contained by the extremes = the rect. contained by the means.

Supposing the same quantity comes in both the 2nd and 3rd places, thus $a : b = b : c$,

$$\frac{a}{b} = \frac{b}{c} \left(e.g. \ \frac{1}{2} = \frac{2}{4}, \ \frac{9}{6} = \frac{6}{4} \right).$$

Then b is called a **mean proportional** to a and c, and c is called a **third proportional** to a and b.

It is plain that finding a third proportional to two numbers, say, 1, 3, is the same as finding a 4th proportional to 1, 3, 3.

Find third proportionals to 3, 4;
4, 3;
5, 6;
6, 5.

It is as well to use the construction in Ex. 4, remembering that if you should get an isosceles triangle by your construction, you have certainly joined the wrong points.

6. Fig. 212 shows a semicircle, whose diameter is AB. The angle BAC is 35°; also CD is drawn perpendicular to AB. Remembering that the angle in a semicircle is a right angle (XXII. 17), you should be able to find in degrees the angles ACD, DCB, CBD.

7. Suppose that in Fig. 212 the angle A were 29°, what would be the number of degrees in each of the other angles, the right angle being as before?

8. You should be able to see that in Fig. 212 the triangles ABC, ACD, CBD are similar. By taking the

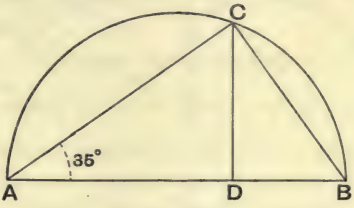


FIG. 212.

sides about the equal angles in order, you will be able to find two other equal ratios for each of the ratios

$$\frac{AC}{CB}, \quad \frac{AB}{AC}, \quad \frac{AB}{BC}.$$

For instance,

Triangle ABC.	Triangle ACD.	Triangle CBD.
$\frac{AC}{CB}$	$= \frac{AD}{DC}$	$= \frac{DC}{DB}$
$\frac{AB}{AC}$	$=$	$=$
$\frac{AB}{BC}$	$=$	$=$

Copy the above table, filling up the blanks.

9. Since $\frac{AD}{DC} = \frac{DC}{DB}$ (Fig. 212), it follows that DC is a mean proportional to AD, DB. Find a mean proportional to 1·6 in. and 0·9 in. in this way. Draw a straight line and mark off on it $AB = 1{\cdot}6$ in., $BC = 0{\cdot}9$ in. (Fig. 213). Bisect AC and make a semi-

circle having AC as diameter. At B draw BD perpen-

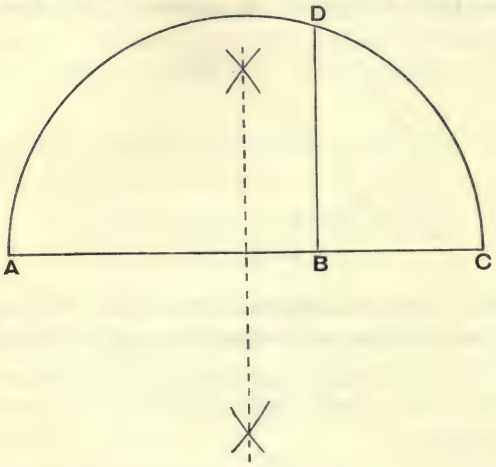


FIG. 213.

dicular to AC meeting the circumference at C. Then BC is a mean proportional to AB and BC. Make this

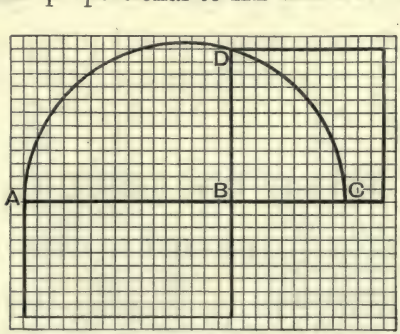


FIG. 214.

construction and measure BD. What is the area of

the square on BD? What is the area of the rectangle contained by AB and BC? Find the areas of the rectangle and square in Fig. 214 by counting the small squares.

10. If x is a mean proportional to a and b, then $\frac{a}{x}=\frac{x}{b}$; also $ab=x^2$, *i.e.* the square on the mean is equal to the rectangle contained by the extremes. Hence, in finding a mean proportional to two lines, say 2 in. and 3 in., we are finding the side of a square of which the area is $3\times 2=6$ sq. in. Construct in this way a square of area 6 sq. in. Measure its side. Also find the square root of 6 to 2 places of decimals. Verify your result by making the construction again on squared paper.

11. Construct, by this method of mean proportionals, squares of area 8 sq. in., 10 sq. in. Measure their sides. Verify your results by repeating the constructions on squared paper.

12. Make a square equal to a rectangle the sides of which are 5 in. and 3 in.

13. In Fig. 215, if LM is a in. long, MN b in. long, then the square on $PM=ab$ sq. in.;

$\therefore$ $PM=\sqrt{ab}$ in. long.

Now suppose that

$a=7$, $b=1$;

then $ab=7$,

and $\sqrt{ab}=\sqrt{7}$.

FIG. 215.

Use this construction to find $\sqrt{7}$ geometrically. You should be able to get the result correct within 1 or 2 in the 2nd place of decimals.

14. Similarly find $\sqrt{3}$, $\sqrt{4{\cdot}7}$ using squared paper.

15. You have found (Ex. 8) that $AD : DC = DC : DB$ (Fig. 216). In other words DC is a mean proportional to AD and DB. But you have also found that

$$AB : AC = AC : AD,$$

i.e. AC is a mean proportional to AB and AD.

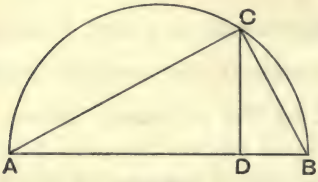


FIG. 216.

What is BC a mean proportional between?

16. Notice carefully, and remember, that in Fig. 217, the upright is a mean proportional between the two parts of the base, and each of the slanting lines is a mean proportional between the whole base and the part immediately underneath the slanting line. For instance, if $AD = 5$, $DB = 2$;

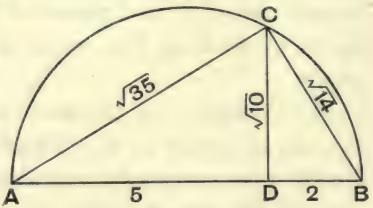


FIG. 217.

then

$$DC = \sqrt{5 \times 2} = \sqrt{10},$$
$$AC = \sqrt{7 \times 5} = \sqrt{35},$$
$$BC = \sqrt{7 \times 2} = \sqrt{14}.$$

Now write down the values of DC, AC, BC, when

(i) $AD = 2$, $BD = 3$,

(ii) $AD = 3$, $BD = 1$,

(iii) $AD = 4$, $BD = 3$,

(iv) $AD = 2{\cdot}5$, $BD = 1{\cdot}5$.

Draw a sketch figure in each case, and write the values against the lines as in Fig. 217, without working out the square roots.

17. Suppose you have to find a mean proportional to two lines 4 in. and 5 in. long. The construction in Ex. 9 may be used, in which case you will have to make a semicircle of diameter 9 in. which is inconveniently large. But if you make $AB = 5$ in., cut off $AD = 4$ in. (Fig. 218), draw DC perpendicular and join AC, then AC is a mean proportional to AB and AD, *i.e.* to 5 in. and 4 in., and your semicircle has a diameter of only 5 in. Use this construction to find mean proportionals to

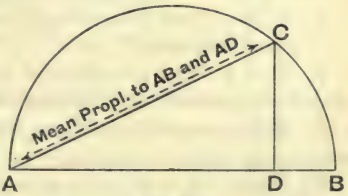


FIG. 218.

(i) 5 in. and 6 in.,
(ii) 12·5 cm. and 8 cm.,
(iii) 7·7 in. and 6·4 in.

18. By the construction in the previous Exercise, find to two places of decimals $\sqrt{15}$, $\sqrt{21}$, $\sqrt{35}$, $\sqrt{8{\cdot}3}$.

ADDITIONAL EXERCISES.

XXVI. a.

1. Draw AB and AC each 3 in. long, making the angle BAC 50°. From B draw BD at right angles to AC. Measure BD and AD in millimetres and thence calculate the value of the ratio $\frac{BD}{AD}$ to two places of decimals.

2. Divide a line 4·8 in. into two parts, in the ratio of 2 : 5.

3. Determine the lengths of third proportionals (greater and less) to lengths of $1\frac{1}{4}$ in. and 2 in.

4. Find a fourth proportional to the lines 1·6 in., 2 in. and 2·5 in., and a mean proportional to the lines 2 in. and 2·8 in.

5. Find a third proportional to the lines 1·8 in. and 2·4 in., and a mean proportional to the lines 2·2 in. and 3·2 in.

6. Find a third proportional to lengths of $\frac{3}{4}$ in. and 1 in., and a mean proportional between lengths of $\frac{3}{4}$ in. and $2\frac{1}{4}$ in.

7. Find a third proportional to (1) lines of $1\frac{1}{2}$ in. and 2 in., (2) lines of 2 in. and $1\frac{1}{2}$ in.

8. Find a third proportional to the lines 3·2 in. and 2·4 in. Also find geometrically the numerical value of $\sqrt{5}$.

9. Make a rectangle of $7\frac{1}{2}$ square inches area ; and reduce it to a square of the same area.

10. Construct a triangle with sides respectively $\sqrt{5}$, $\sqrt{7}$, and $\sqrt{2}$ inches obtained geometrically.

11. Construct a line of length $\sqrt{5}$ inches. On it describe a square, and in the square inscribe 4 equal semicircles with their diameters adjacent.

12. Make a square of area 10 square inches. Divide it into 5 equal parts by straight lines drawn from the middle point of one of the sides.

13. Make a square of area 12 square inches, and trisect it by straight lines drawn from the middle point of one side.

14. Inscribe a square and an equilateral triangle in a circle of 1·73 inches radius, and find *by construction*, and write down, the ratio that the area of the square inscribed in the circle bears to the area of the square described on one of the sides of the equilateral triangle.

15. With your instruments make a triangle ABC having $AB = 13{\cdot}5$ cm., $BC = 11{\cdot}4$ cm., $CA = 17{\cdot}3$ cm. In AB take any 2 points P and Q, and draw PP' and QQ' parallel to BC to cut AC in P' and Q'. Measure AP, PP', etc., and calculate the ratios $AB : BC$, $AP : PP'$, $AQ : QQ'$ to the nearest hundredth. What conclusion is suggested by your results? Prove the general truth of the conclusion, referring to Euclid's or some other text book of Geometry.

CHAPTER XXVII.

AREAS OF SIMILAR FIGURES.

1. Two triangles are similar when the angles of one are severally equal to the angles of the other. The triangles must then be of the same shape and have their corresponding sides in the same ratio. But when a figure has more than 3 sides, its shape does not depend alone on the angles between the sides. For instance, two quadrilaterals are shown in Fig. 219,

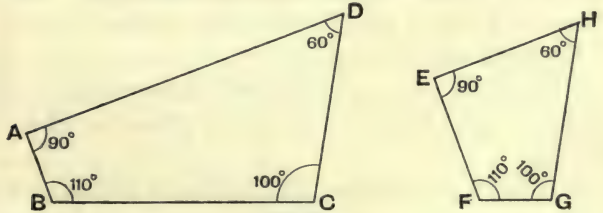


FIG. 219.

having the angles of one respectively equal to the angles of the other, but the shapes of the figures are quite different, one being low and broad, the other high and narrow. If two such figures are to be similar, their corresponding sides must be proportional, and it is plain that this is not the case in Fig. 219. Take the sides containing the angles of 110° for instance. Obviously

$\frac{AB}{BC}$ is not equal to $\frac{EF}{FG}$, and if the diagonals AC and EG are drawn, it is plain that the angles ACB, EGF are not equal.

Fig. 220 shows two five-sided figures which are similar, and it is plain that the triangles into which

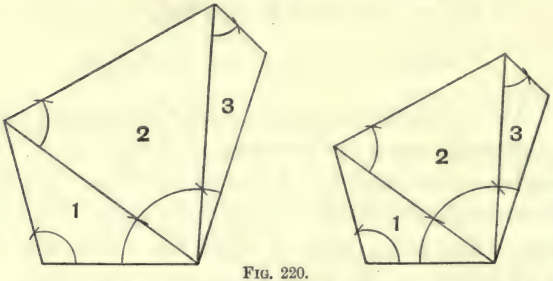


FIG. 220.

they are divided are similar, taking them in the numbered order. Measure the marked angles, and on a base of 2 inches construct another similar figure:

(i) using your protractor;

(ii) making the angles of your figure equal to the marked angles by the construction given in XIV. 23, 24.

2. To find out whether all squares are similar or not, draw two squares of different size. Divide each square into two triangles and measure the angles of each triangle. Mark the number of degrees against each angle.

3. It is possible to write down the angles of the halves of a square without measuring them, simply by using what has been found out about triangles in Chap. XIII. $ABCD$ (Fig. 221) is a square, and

AC is its diagonal. State the number of degrees in each of the angles marked, giving reasons for your statements.

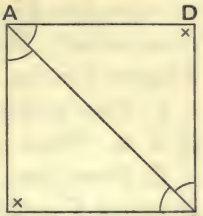


FIG. 221.

4. Fig. 222 shows a shaded square forming part of another with side twice as long. It also shows 2 **similar** triangles, the shaded triangle having sides half as long as those of the large triangle. How many times does the large square contain the shaded square? How many times does the large

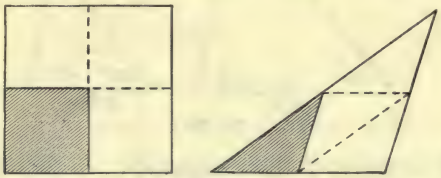


FIG. 222.

triangle contain the shaded triangle? Answer the same question for Fig. 223.

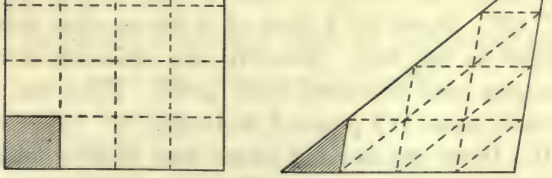


FIG. 223.

Draw a figure showing the comparative area of squares whose sides are in the ratio of 5 : 1. Do the same thing for triangles.

5. A triangle has sides 3, 4, 5 cm. A similar triangle has its shortest side 1·5 cm. What are the other sides? How many times is the smaller triangle contained in the larger?

6. Two similar triangles have bases in the ratio of 3 : 2. What is the ratio of their areas?

7. Draw two similar triangles of the dimensions shown in Fig. 224. Measure their altitudes (XVIII. 14)

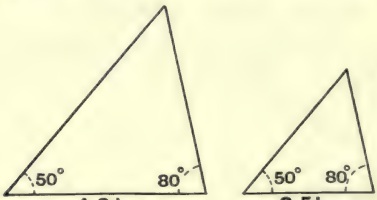


FIG. 224.

and find the ratio of their altitudes. It should be 6 : 5. Why?

8. Draw any two similar triangles, and find by measurement whether their altitudes are proportional to their bases.

9. Draw a triangle, base 9 cm., angles at base 44°, 64°. Draw on a base of 6 cm. another triangle similar to the first. Measure the altitudes of these triangles and calculate their areas. Find the ratio of their areas to 1 place of decimals.

10. Draw on squared paper two similar triangles with their bases in the ratio of 4 : 3. Find the ratio of their areas. Mark out squares on the bases of the two triangles. Is the ratio of the squares the same as the ratio of the areas of the triangles?

11. The area of a triangle is $\frac{1}{2}$ base $\times$ altitude;
$\therefore$ the area of ABC (Fig. 225) $= \frac{1}{2}BC \times AD$.

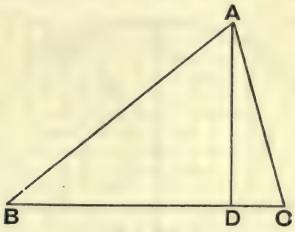


FIG. 225.

Now $$\frac{AD}{AB} = \sin B;$$

$$\therefore AD = AB \times \sin B;$$

$$\therefore \text{area} = \tfrac{1}{2} BC \times AB \times \sin ABC$$

$$= \tfrac{1}{2} ac \sin B \text{ (XXV. 12).}$$

Find the area of a triangle, two sides of which are 3 in. and 2 in., and the angle between them 60° ($\sin 60° = 0{\cdot}866$). Verify your result by constructing the triangle and calculating its area from measurement.

12. Construct a triangle from the following data:

$$a = 6 \text{ cm.}, \quad b = 4 \text{ cm.}, \quad \angle C = 67°,$$

and another having

$$a = 9 \text{ cm.}, \quad b = 6 \text{ cm.}, \quad \angle C = 67°.$$

Measure the other angles and side of each triangle. Are the triangles similar? Find in two ways, (i) by measurement, (ii) by using the formula in Ex. 11, the ratio of the areas of the triangles.

13. In Fig. 226 a circle is shown drawn on squared paper. Find, by counting the squares, the ratio of the area of the circle to the area of the square on the

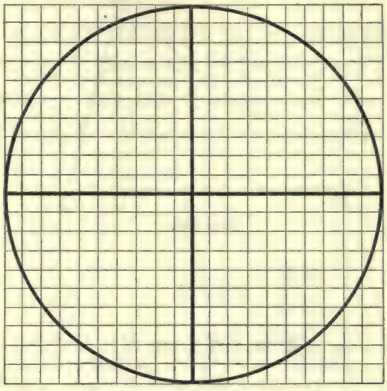


FIG. 226.

radius. Repeat this with Fig. 227 in which a circle of a different radius is shown, drawn on squared paper

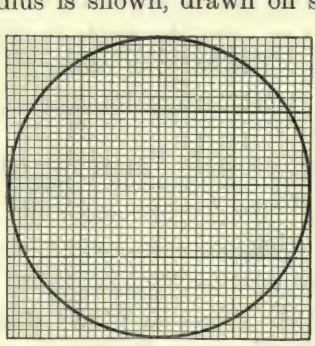


FIG. 227.

of different sized squares. (Remember the instructions given in VII. 4.)

14. Draw for yourself on squared paper a circle of any convenient radius, and find as nearly as you can the ratio of its area to the square on its radius.

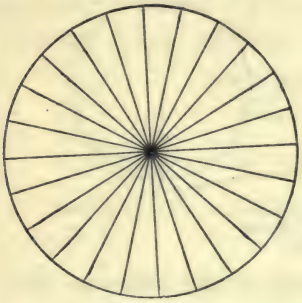


FIG. 228.

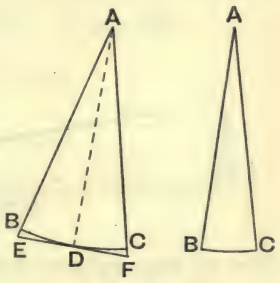


FIG. 229.

15. Fig. 228 is a circle divided into **sectors** (XVI. a. 9). The area of the circle is the sum of the areas of all the sectors. Now the area of the sector ABC (Fig. 229) is very nearly the same as that of the triangle AEF which is $\frac{1}{2} AD \times EF$, and EF is very nearly the same length as the arc BC. The smaller we make the angle BAC, the more nearly does the arc BC approach to the straight line EF. So the area of a sector of very small angle may be taken as

$$\frac{1}{2} AD \times BC = \frac{1}{2} \times \text{radius} \times \text{arc}.$$

Now the circle

$= \text{sector} + \text{sector} + \text{sector} + \text{etc.}\ldots$ all round the circumference

$= \frac{1}{2}$ radius $\times$ {arc + arc + arc + ... all round the circumference} (XXVIII. 6.)

$= \frac{1}{2}$ radius $\times$ circumference

because the arcs added up make the whole circumference.

So the area of a circle is the same as the area of a triangle whose height is the radius of the circle and whose base is its circumference (Fig. 230), and the

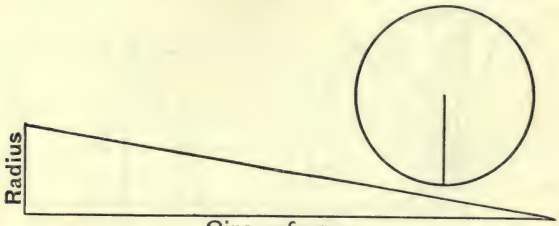


FIG. 230.

area of any sector of a circle is the same as that of a triangle whose height is the radius of the sector and whose base is the arc of the sector (Fig. 231).

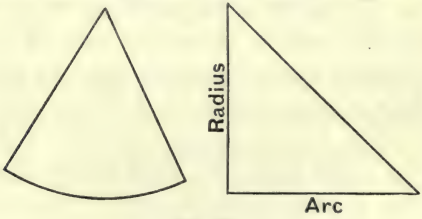


FIG. 231.

Now you have found that the circumference of a circle is about 3·14 times its diameter. Draw on squared paper a circle of radius 2 inches and find its area in square inches by counting squares and by calculation.

16. The ratio of the circumference to the diameter of a circle is not exactly 3·14. 3·14159 is the value correct to 5 places; but even this is not exact. When its exact value is to be taken, the Greek letter π is

used. Then, when you want to work out a numerical result, you can put, instead of π, $3\frac{1}{7}$ or 3·14 or 3·1416 or 3·14159265 according to the degree of accuracy required. For most purposes the first two of these values are near enough. Now, since

$$\frac{\text{circumference}}{\text{diameter}} = \pi,$$

$$\therefore \text{ circumference} = \pi \times \text{diameter}$$

$$= \pi \times 2r \text{ (where } r \text{ is the radius)}$$

$$= 2\pi r.$$

$$\therefore \text{ area of a circle} = \tfrac{1}{2}r \times 2\pi r = \pi r^2.$$

In words, it may be said that the area of a circle is about 3·14 times the square on its radius. If the radius of a circle is 10 cm., what is its area? In counting the squares in Figs. 226, 227 you have probably made mistakes in estimating the fractions of squares. Find the correct results by multiplying the area of the square on the radius by 3·1416.

17. Find the ratio of a semicircle to the square on

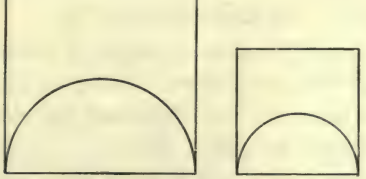


FIG. 232.

its diameter. Are the two semicircles in Fig. 232 proportional to the two squares?

18. Find the ratio of the areas of the two equilateral triangles in Fig. 233. Is it the same as the ratio of the squares?

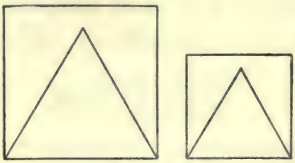


FIG. 233.

19. In the right-angled triangle ABC (Fig. 234)

$$a^2 = b^2 + c^2;$$

$$\therefore\ \pi a^2 = \pi b^2 + \pi c^2;$$

$\therefore$ the area of the circle whose radius is BC is equal to the sum of the areas of the circles whose radii are AC, AB.

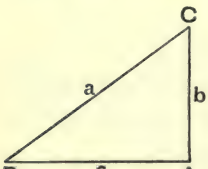


FIG. 234.

Describe a circle of area equal to the sum of the areas of two given circles.

20. Describe a circle of area equal to the difference of the areas of two given circles.

21. Make an equilateral triangle of area equal to the sum of two equilateral triangles of which the sides are 6 cm. and 8 cm. Measure its side.

22. Make an equilateral triangle of area equal to the difference of two equilateral triangles of which the sides are 5·2 in. and 2 in. Measure its side. Employ the method given in XXII. 24.

23. Draw two similar figures of any shape (Fig. 220) on squared paper and show by counting squares that their areas are proportional to the squares on their bases.

24. If $ABCDE$, $abcde$ (Fig. 235) are similar figures, then $\frac{AB}{BC}=\frac{ab}{bc}$ and so on; and AB, ab are **corresponding** sides, or as Euclid calls them, **homologous** sides. Similarly BC, bc are homologous; so are CD, cd. And if AB is double of ab, BC is double of bc, and so on. But the **area** of the figure $ABCDE$ is not double, but **four times** that of $abcde$. Again if $AB=3$ times ab, then each side of $ABCDE$ is 3 times the corresponding or homologous side of $abcde$; and the area of $ABCDE$ is 3^2 $(=9)$ times the area of $abcde$.

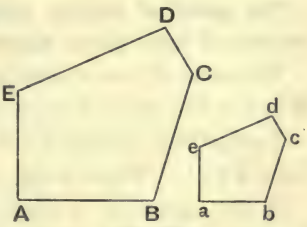


FIG. 235.

Again, if the radius OA of the circle ABC is 4 times oa (Fig. 236), then the diameter and circumference of

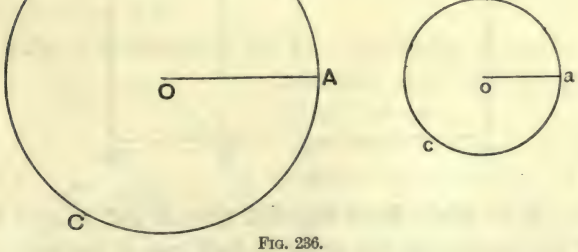


FIG. 236.

ABC are respectively 4 times the diameter and circumference of abc. This is expressed by saying that the **linear dimensions** of the two figures are in the

ratio 4 : 1; but the area of the larger circle is 16 times (4^2) the area of the smaller circle. The same is true of all similar figures in whatever ratio their linear dimensions may be; and the fact is stated thus:

The areas of similar figures are proportional to the squares of their linear dimensions.

The area of one circle is 25 times that of another. What is the ratio of their radii?

Equilateral triangles are made having sides in the ratio 2·7 : 1. What is the ratio of their areas?

If one square be 3 times as large as another square, find the ratio of their sides to 2 places of decimals.

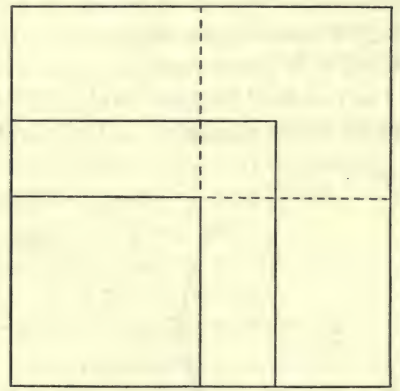


FIG. 237.

25. It is plain from Fig. 237 that if AC is half of AB, the square on AC is not a half but a quarter of the square on AB. So that the side of the square which is half of the square on AB (or double of the square on AC) must be greater than AC and less

than AB. AD is about the right length. Verify by copying on squared paper and counting squares, that the square on AD is about half of the square on AB.

Since $$\frac{AD^2}{AB^2}=\frac{1}{2};$$

$$\therefore \frac{AD}{AB}=\frac{1}{\sqrt{2}}.$$

Also since $$AD^2=2AC^2,$$

$$\frac{AD}{AC}=\sqrt{2}.$$

Now $\sqrt{2}$ is a mean proportional between 1 and 2; $\therefore$ AD is a mean proportional between AC and AB. Fig. 238 shows the construction for obtaining AD.

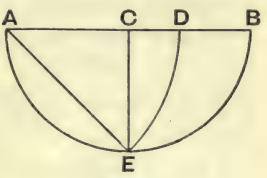


FIG. 238.

On AB make a semicircle. At C draw CE perpendicular to AB. Then AE is a mean proportional to AC and AB (XXVI. 16). Mark off AD equal to AE.

Make a square, side 2·3 in. Construct a square of twice the area, and another of half the area.

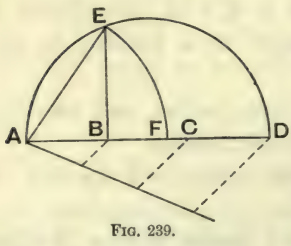


FIG. 239.

26. Make a square of side 2 inches and another of 3 times its area (Fig. 239).

The square on AE is 3 times the square on AB, and $\frac{1}{3}$ of the square on AD. Why?

Between what lines is AF a mean proportional?

27. In Fig. 240 the line AC is $\frac{3}{4}$ of AB. CD is drawn perpendicular to AB. What is the ratio of the squares on AC and AB? What is the ratio of the squares on AD and AB? What is the ratio of the squares on AC and AD? Verify your answers by drawing the figure on squared paper.

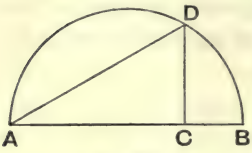


FIG. 240.

28. ABC is a triangle. AC is trisected at D and E. Perpendiculars DF, EG are drawn to meet the semicircle described on AC. AH, AK are marked off equal to AF, AG. LH, MK are drawn parallel to BC.

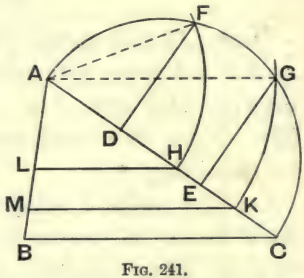


FIG. 241.

The triangles ALH, AMK are similar to ABC. Give reasons for the statement that the area of the triangle ALH is $\frac{1}{3}$ of ABC, and that AMK is $\frac{2}{3}$ of ABC.

Apply this method to the following problem:

Construct a triangle sides 3, 4, 5 in., and divide it into 3 equal parts by straight lines parallel to the shortest side.

29. Make a circle of 1 inch radius, and another of 4 times its area.

30. Make a circle of 7 cm. radius, and another of half its area.

31. Make a triangle, area 3 square inches (XVIII. 14).

32. Make a triangle having an angle of 60°, area 3 sq. in. (Fig. 242).

33. If x is a mean proportional between a and b then $x^2 = ab$ (XXVI. 10). But the area of the triangle ABC (Fig. 242) is $\frac{1}{2}ab\sin 60°$. If we make an isosceles triangle with one angle 60° and the sides containing it each equal to x, its area will be $\frac{1}{2}x^2\sin 60°$. And if $x^2 = ab$, this will be the same as the area of the first triangle, viz. 3 sq. in. What are the remaining angles of an isosceles triangle whose vertical angle is 60°? What sort of triangle is it?

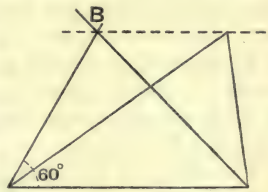


FIG. 242.

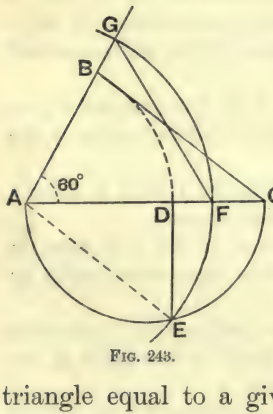


FIG. 243.

Apply this to making an equilateral triangle, area 3 sq. in. (See Fig. 243 in which AF is a mean proportional to AB and AC, and the triangle AFG is equal in area to the triangle CAB.)

34. Make an equilateral triangle equal to a given square.

ADDITIONAL EXERCISES.

XXVII. a.

1. Construct a triangle with base $2\frac{1}{2}$ inches, height 2 inches, vertical angle 60° (XXIV. 10). Bisect it by a straight line drawn parallel to the base.

2. On a base 3 in. describe a triangle having vertical angle 60° and one other angle 50°. Make an isosceles triangle equal to it, and then reduce it to a square of the same area.

3. Construct a polygon $ABCDE$ having each side 2 in., and having the angles of A and B 108° and those at C and E 36°. Reduce the figure to a square of the same area.

4. Describe a regular pentagon on a base 2 in. : and make a similar figure of half the area.

5. Draw an isosceles triangle with perimeter 10 in. and its sides double the base.

Cut off $\frac{1}{3}$ of this triangle by a line parallel to the base. Bisect the quadrilateral figure remaining, by a line drawn through the extremity of the base.

6. In a square, side 2·8 in., inscribe a second square with area two-thirds that of the other.

7. Draw an isosceles triangle, base 2 in. and vertical angle 47°, and divide it into four equal parts by lines drawn parallel to the base.

8. Make a triangle with sides of 2, $2\frac{1}{2}$, 3 inches, and divide it into three equal parts by lines drawn parallel to one of the sides.

9. Make a pentagon on a base 2·8 inches and in it place a second with area $\frac{2}{3}$ that of the original pentagon.

10. Draw a triangle with perimeter 10 in. and sides in the ratio of 2 : 4 : 5, and divide it into three equal parts by lines parallel to the shortest side.

11. Construct a pentagon side $1\frac{1}{2}$ in. and another three-fifths of the first. Construct a rectangle equal to four-fifths of the latter and find its area in square inches.

12. Construct an isosceles triangle on a base of 3 in., having its vertical angle 40°. Construct also a similar one of half the area.

13. Convert a given rhombus containing an angle 40° into an equilateral triangle of the same area.

14. Reduce a square with diagonal $2\frac{1}{2}$ inches to an equilateral triangle of the same area.

15. On a line 1·75 inches long describe by means of your

protractor, or in any way you please, a regular heptagon, and construct an equilateral triangle equal in area to $\frac{1}{3}$ of the heptagon.

16. The angles of a triangle are in the ratio of 2, 3, 4, and the perimeter is 7 in. Construct the triangle, and make an equilateral triangle equal to it.

17. In a circle, radius 2·5 in., inscribe a regular heptagon, and make an equilateral triangle equal to it in area.

18. Construct a square $ABCD$ and an equilateral triangle ABE on the same side of a base AB 3 in. long. Reduce the space between them to an equilateral triangle.

19. Construct a regular octagon of area 8 square inches.

20. On a base of 2 in. make a square ; on the three other sides make equilateral triangles. Make another square equal to the whole figure thus formed.

21. In a circle, radius 2·7 in., inscribe a regular heptagon. Find its area in square inches.

CHAPTER XXVIII.

MENSURATION FORMULAE.

1. If two lines having lengths of 3 inches and 2 inches are placed end to end, the length of the whole line is 5 inches. What is the total length of a line made by an inch and a centimetre placed end to end? (Chap. II.)

2. If we wish to speak of the length of a line which has not been measured, we can use a letter instead of a number, only it must be clearly understood what unit of measurement is being used. Thus a line may be a inches or x centimetres. Two lines, a and b inches, added together make a line having

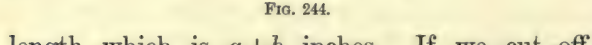


FIG. 244.

a length which is $a+b$ inches. If we cut off y centimetres from a line x centimetres long, there remains $x-y$ centimetres. In Fig. 244,

$$AB=a.$$
$$BC=b.$$
$$CD=c.$$
$$ED=x.$$

What is the length of AE? of BE?

3. If a inches and b centimetres are added together, what is the length of the whole line?

4. The small squares in Fig. 245 are $\frac{1}{10}$ inch each way. There are 100 of them in every square inch. If we measure in tenths of an inch, the figure $ABCD$

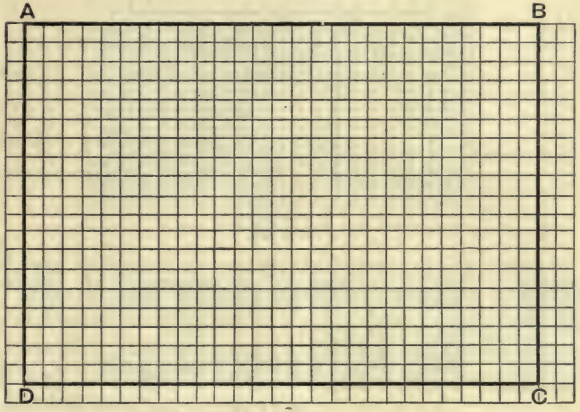


FIG. 245.

is 27 by 19, and it contains 27×19 of the small squares. If we measure in inches, the figure is 2·7 by 1·9 and it contains $\dfrac{27 \times 19}{100}$ $(= 2{\cdot}7 \times 1{\cdot}9)$ square inches.

How many small squares does each rectangle in Fig. 246 contain? What is the area of each in square inches? (Answer in decimals.)

5. If our measurements of length are taken to two places of decimals, this will require us to divide each of the small squares in Fig. 245 into 100 smaller

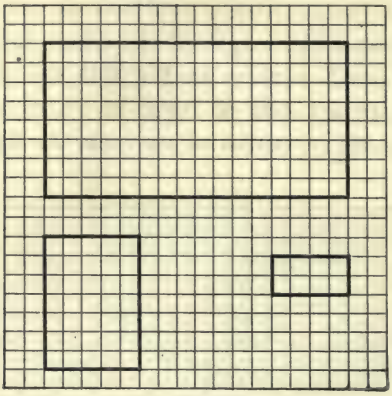


FIG. 246.

squares. Thus, the area of a square 2·54 cm. each way is

$$= \frac{254 \times 254}{100 \times 100} = 2{\cdot}54 \times 2{\cdot}54 \text{ sq. cm.}$$

$$= 6{\cdot}4516 \text{ sq. cm.}$$

There is no limit to the number of places of decimals to which the calculations can be taken except the limits of the possible accuracy of the measurement. It is difficult to measure accurately to more than 2 places of decimals of an inch. Now suppose a rectangle is a inches long and b inches broad. We agree to call its area ab square inches, where ab means $a \times b$. Fig. 247 shows a rectangle $(a+b)$ long and

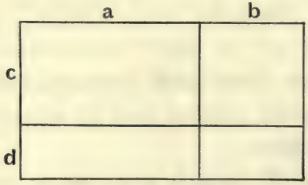


FIG. 247.

$(c+d)$ broad. Its area is $(a+b)(c+d)$. What is the area of each part of the figure?

6. If one side of a rectangle be made up of a number of parts a, b, c, d, the whole side is

$$a+b+c+d.$$

If the breadth of the rectangle be x, the area is $x(a+b+c+d)$. It is plain from Figure 248 that this area is made up of the areas ax, bx, cx, dx. Hence we find that

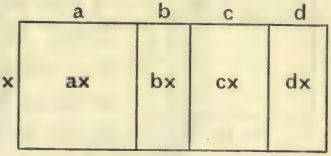


FIG. 248.

$$x(a+b+c+d)=ax+bx+cx+dx.$$

This fact is stated in words by Euclid (II. 1) thus:

If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the sum of the rectangles contained by the undivided line and each of the parts of the divided line.

State algebraically the following facts corresponding to Euclid II. 2, 3, 4. (Remember that aa is written a^2.)

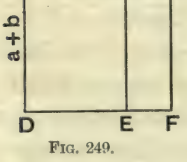


FIG. 249.

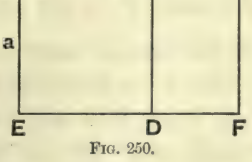


FIG. 250.

$ADFC=ADEB+BEFC.$ (Fig. 249.)

$AEFC=AEDB+BDFC.$ (Fig. 250.)

The square on FG = the sum of its four parts (Fig. 251).

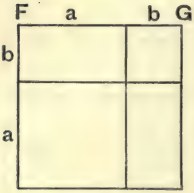


FIG. 251.

7. In Fig. 252, what is the area of the large square? (Chap. XX.) What is the area of each of the triangular portions? What remains when they are subtracted? What is the length of the side of the shaded square in the figure? What is its area? What formula does this figure prove?

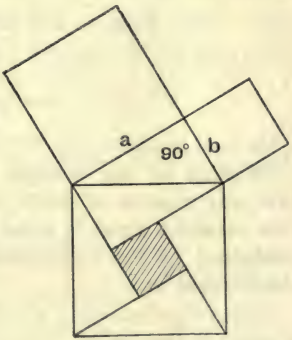


FIG. 252.

8. Write down in the shortest way possible the following areas. (State the unit of area in each case.)

(i) The area of a rectangle of which the length is $(a+b+c)$ feet and breadth is a yards.

(ii) The area of a triangle of which the base is x cm. and height y cm.

(iii) The area of a circle of which the radius is r feet $\left(\text{use the letter } \pi \text{ for the ratio } \frac{\text{circumference}}{\text{diameter}}\right)$.

9. If the area of each small square of the base of a block like A in Fig. 253 is 1 sq. cm., and the slabs are 1 cm. thick, how many cubic centimetres are there in a single slab of the block? How many cubic centimetres are there in three of the slabs? How many cubic centimetres are there in the whole of the slabs?

What is the number of cubic centimetres in a block a cm. long, b cm. broad, c cm. thick?

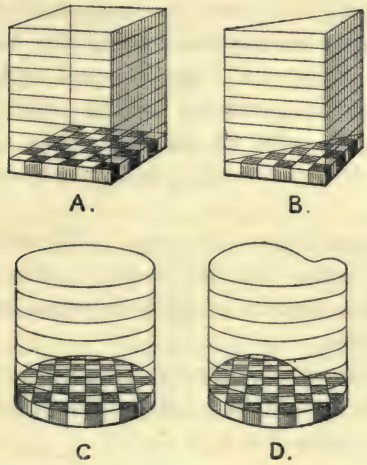


FIG. 253.

10. Now examine the block B (Fig. 253), and determine (1) the area of the base, (2) the number of cubic centimetres in a single slab, (3) the number of cubic centimetres in the whole of the slabs, that is, the volume of the triangular prism.

What is the volume of a **triangular prism** (*i.e.* a figure like B) having as base a triangle the area of which is $\frac{1}{2}ab$ sq. cm. and the height h cm.?

Proceed in the same way to determine the volumes of blocks C and D in Fig. 253.

What is the volume of a **cylinder** (*i.e.* a figure like C; a *roller* is the literal meaning of the word),

the height of which is h cm. and the base a circle of radius r cm.

What is the volume of any prism or cylinder of which the height is h and the area of the base A ?

11. Draw a figure illustrating the fact that the area of the walls of a room is equal to the rectangle contained by the perimeter and the height.

What is the area of the four walls of a room of which the length, breadth, and height are l, b, h feet respectively ?

12. Cut out a piece of paper to fit exactly round a cylinder. Notice that its length is the same as the circumference, and its breadth the same as the height of the cylinder.

What is the area of the curved surface of a cylinder of which the height is h inches and the base a circle of radius r inches ?

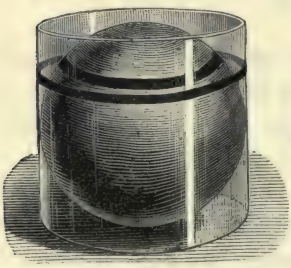


FIG. 254.

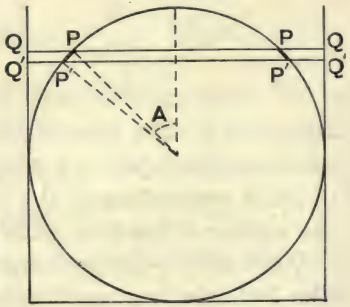


FIG. 255.

13. Fig. 254 represents a sphere just fitting inside a hollow cylinder. Fig. 255 shows a narrow belt on the sphere and the narrow belt corresponding to it on the inside surface of the cylinder. You may notice

that the belt of the cylinder is the longer of the two; but the belt of the sphere is the broader. And it is not hard to satisfy yourself that the ratio

$$\frac{\text{length of belt of sphere}}{\text{length of belt of cylinder}} = \frac{\text{breadth of belt of cylinder}}{\text{breadth of belt of sphere}},$$

and that each ratio is equal to the sine of the angle marked A in the figure. Now the area of each belt (if narrow) is length × breadth. So we find that the two belts are of the same area.

The same is true for all corresponding belts: and by adding all the belts together, we find that the area of the surface of the sphere is equal to the area of the *curved* surface of the cylinder. Now the height of the cylinder is the diameter of the sphere, and the radius of the cylinder the same as the radius of the sphere. Now find the area of the surface of a sphere whose radius is r. Notice that it is 4 times that of a certain circle. What circle?

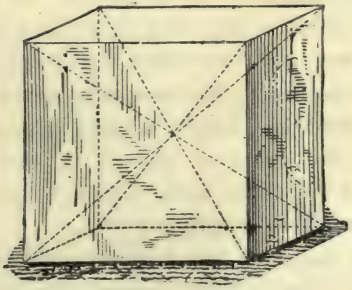


FIG. 256.

14. A cube may be formed by six **pyramids** (Fig. 256). Suppose another cube of the same size to be cut into

six slabs as in Fig. 257. Then the volume of one of the pyramids is equal to the volume of one of the slabs, for each is one-sixth the volume of the cube. The volume of a slab is equal to the base multiplied by the height of layer, which is one-third the height of a pyramid. Therefore the volume of a pyramid is equal to the base multiplied by one-third the height.

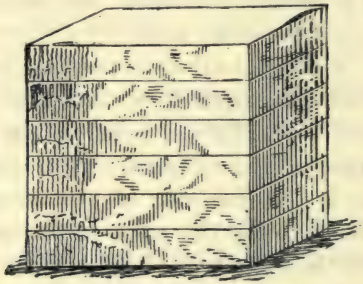


FIG. 257.

This rule applies to every pyramid. In other words, a pyramid could be flattened down to a block having the same base and one-third the height. Find the volume of a pyramid, the base of which is a square of side a feet, and the height h feet; also of a pyramid of height h cm., area of base A sq. metres; also of a **cone** (*i.e.* a pyramid with a round base) of height h cm., base a circle of radius r cm.

15. Place together two triangular blocks or prisms to form a cube. Then place the shaded block against the unshaded one so as to form the slant shape shown in Fig. 258. This shape is called a **parallelepiped**. Evidently the blocks have the same

total volume whether they are placed to make a cube or a sloping parallelepiped; hence the cubic

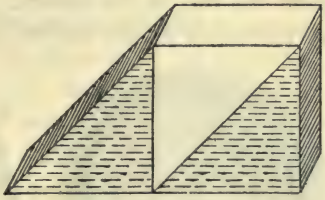


FIG. 258.

content or volume of a parallelepiped, like the volume of a cube, is equal to **the area of the base multiplied by the perpendicular height.**

The same is true for slanting cylinders, *e.g.* volume of cylinder in Fig. 259 = area of base × perpendicular

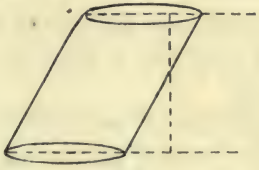


FIG. 259.

height. Give the corresponding formula for volumes of slanting pyramids and cones (Fig. 260).

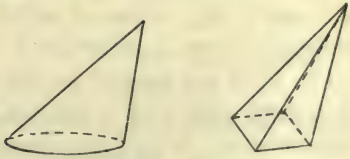


FIG. 260.

16. Just as a circle may be cut into small sectors which may be regarded as triangles (XXVII. 15), so a

sphere may be cut into "**solid sectors**," each of which may be regarded as a pyramid. Fig. 261 shows one of these pyramids cut out of its sphere. The height of each of these pyramids is the radius of the sphere.

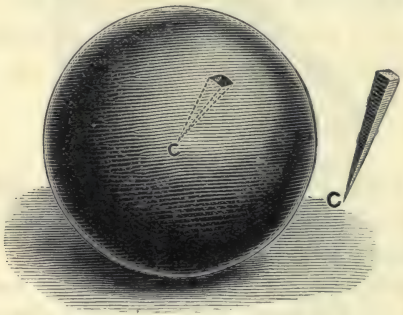


FIG. 261.

Now just as the circle was the sum of all the triangles, so the volume of a sphere is the sum of the volumes of all the pyramids

$= \frac{1}{3}$ height × area of base + etc.

$= \frac{1}{3}$ height × {sum of all the bases}

$= \frac{1}{3}$ radius × area of surface of sphere.

FIG. 262.

You have found the area of the surface (Ex. 13). Now write down the volume of a sphere the radius of which is r centimetres.

17. Find the area of the curved surface of a **right cone** (not a slanting cone) the base of which is a circle of radius r centimetres and the slant side of which is l centimetres. (Divide the curved surface into triangles like the circle in XXVII. 15.) (Fig. 262.)

Hence find the area of the curved surface of a cone the height of which is h and the radius of the base r (Fig. 263).

18. The **semi-vertical angle** of a cone is shown in Fig. 263. The cone may be regarded as the solid figure described by the right-angled triangle ABC revolving round the side AB as axis. BAC is the semi-vertical angle. Let us call it α.

FIG. 263.

Then $$\frac{r}{h} = \tan \alpha;$$

$$\therefore\ h = \frac{r}{\tan \alpha}.$$

$\therefore$ volume of cone $= \frac{1}{3}\pi r^3 \div \tan \alpha$.

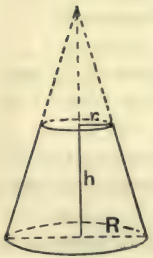


FIG. 264.

Fig. 264 shows a **frustum** of a cone, that is a cone with its top cut off. The top is shown by dotted lines in the figure. Prove for yourself by regarding the frustum as the difference of two cones that the volume of a frustum, the height of which is h and the radii of top and bottom r and R respectively, is $\frac{1}{3}h(R^2 + Rr + r^2)$.

ADDITIONAL EXERCISES.

XXVIII. a.

1. Compare the area of an equilateral triangle with that of a circle which circumscribes it.

2. A well 8 feet in diameter and 35 feet deep has to be lined with bricks 9 ins. thick. Find the weight of the bricks in tons if a brick 9 in. $\times$ 4½ in. $\times$ 3 in. weighs 5 lbs. ($\pi = \frac{22}{7}$).

3. How would you find out by experiment how many times the diameter of a circle is contained in the circumference?

4. A right-angled triangle has a hypotenuse 10 in. long, and another side 6 in. long. Find the areas of the circumscribed and inscribed circles in square inches correct to three places of decimals. [Take $\pi = 3{\cdot}1416$.]

5. A square and regular hexagon are described about a given circle. Compare their areas.

6. Find the whole surface and volume of a solid cylinder whose length is 7 in. and diameter of base 6 in. ($\pi = 3\frac{1}{7}$.)

7. Prove that the length of the diagonal of a cube is $\sqrt{3}$ times the length of the edge of the cube. (IX. 7.)

8. The sides of a triangular prism are 3, 4 and 5 inches, and the length 15 inches: find the number of cubic inches in the volume.

9. Write down formulae for the area of a circle, the surface of a sphere, and the surface of a right circular cone.

A cubic foot of brass is drawn into a wire of one-tenth of an inch in diameter: find its length.

10. A solid cylinder of length 8 in. and radius 6 in. is equal in volume to a sphere. Find the whole surfaces both of the sphere and cylinder.

11. A hemispherical bowl has inner radius 6 in. and the material is 1 in. thick. Find the weight of the bowl at 2 ounces to the cubic inch.

12. If the ratio of the perimeter of a circle to its diameter is 3·1416 to 1, find the volume and the surface of a cylinder, the radius of which is 4 feet and the height 100 feet.

13. Water flows in a V-shaped gutter. Find how the volume discharged in a day depends on the depth of the water and the speed of flow.

What is the daily discharge in cubic metres when the depth is 20 centimetres and the speed a metre per second, the sides of the trough meeting in a right angle? (*Navy.*)

14. Give two methods of finding out the volume of a cylinder. Sketch the apparatus you would use.

15. Cut out a piece of paper which will just cover the curved

surface of a given cylinder. The paper will be rectangular in shape, and its base will be equal to the circumference of the cylinder, while its height equals the height of the cylinder. Find the area of the paper, then find the area of the curved surface of the cylinder by multiplying the circumference by the height, and compare the two results.

This exercise proves the rule which states that, to find the area of the curved surface of a cylinder it is necessary to multiply the circumference of the base by the height of the cylinder. You also know that the area of each circular end is equal to the square of the radius multiplied by $3\frac{1}{7}$. Use this knowledge to work out the following :

(*a*) Find the area of the curved surface of a cylinder 10 in. high and 2 in. diameter.

(*b*) Find the area of the whole surface of a cylinder, with plane ends, 6 ft. high, and 2 ft. radius.

(*c*) How many square feet of paper would be required to cover the curved surface of a cylindrical pillar 20 ft. high and 2 ft. in diameter ?

16. A rectangle, whose sides are 3 inches and 4 inches, is made to revolve about its adjacent sides in turn. Find the volumes of the cylinders thus formed.

Find the radius of a sphere equal in volume to their difference.

17. A boiler has the form of a right circular cylinder with two convex hemispherical ends ; show that the area of its external surface is equal to the product of its greatest length and the circumference of the circular section of the cylinder.

18. Find the number of cubic feet of earth removed per yard length from a cutting in level ground 12 feet in depth, the breadth of the base of the cutting being 15 feet, and the slopes of both sides 45°.

19. The great pyramid of Egypt was 481 feet high, when complete, and its base was 764 feet in length. Find the volume to the nearest whole number of cubic yards.

20. The base of a pyramid covers a square of $13\frac{2177}{4415}$ acres, and its height is 480 feet. Find a side of the square and the volume of the pyramid.

21. Find the volume and surface of a cone whose height is 12 in. and diameter of base 10 in.

22. A solid sphere of radius 6 in. is melted and drawn out into a long cylindrical wire of thickness $\frac{1}{8}$ inch. Find the length of the wire in yards.

23. A spherical shell of iron whose diameter is 1 foot is filled with lead. Find the thickness of the iron when the weights of iron and lead are equal. A cubic inch of iron weighs 4·2 oz., and a cubic inch of lead weighs 6·6 oz.

24. A uniform trench is dug 4 metres long and 86 cm. deep; the sides are sloping so that the trench is 1 metre wide at the top and 72 cm. at the bottom. Find to the nearest kilogram the weight of the material removed, if a cubic metre of it weigh 2000 kilograms.

25. The cross section of an iron wire is 1 square millimetre. A cubic centimetre of the wire weighs 7·9 grams, and the force required to break the wire asunder is the weight of 35 kilograms. What length of the wire suspended by one end will just break under its own weight?

CHAPTER XXIX.

EUCLID III. 35, 36. QUADRATICS.

1. Draw a circle of 5 cm. radius. Take a point E 2 cm. from the centre O (Fig. 265). Through E draw any two chords AEB, CED. Measure in cm. and decimals AE, EB and find their product, (or the area in sq. cm. of the rectangle contained by AE and

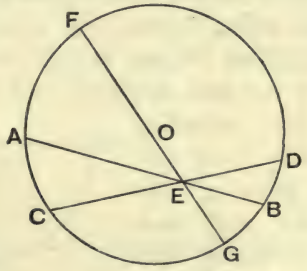


FIG. 265.

EB.) Do the same for CE and ED. If your work has been accurate, the product in each case should be 21 sq. cm. Now draw the diameter FEG.

$$FE = 7 \text{ cm.} \quad EG = 3 \text{ cm.}$$

What is the rectangle contained by FE and EG?

2. Repeat the above with a circle of 3 inches radius, the point E being taken 1 inch from the

centre. What is the area of the rectangle contained by the two portions (or, as Euclid calls them, **segments**) of each chord?

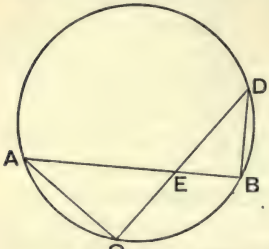


FIG. 266.

3. In Fig. 266 the triangles AEC, DEB are similar. State which are the equal angles (XXIV. 7). Obviously the angles at E are equal. If the triangles are similar, the sides about the angle AEC are proportional to the sides about the angle DEB. Write down the proportion.

What two rectangles are equal? (XXVI. 5.)

4. In Fig. 267, the radius of the circle is 5 cm. $OC=3$ cm. DCE is drawn at right angles to AB. Then $DC=CE$. (Why? See Chap. XXII.) Find DC by making use of the fact discovered in the preceding Exercise. Verify your calculation by drawing the figure carefully and measuring DC. (Compare Euc. II. 5.)

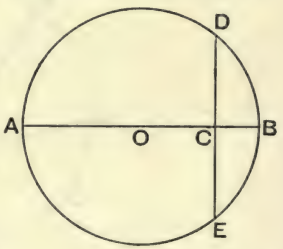


FIG. 267.

5. In Fig. 267 DC is a mean proportional between two lines. Which two lines? Write down the proportion, and compare it with the equation given by Ex. 3.

6. In a circle of 2·5 inches radius, chords are drawn through a point 0·5 inches from the centre. What is the area of the rectangle contained by the segments of any one of the chords?

7. Draw a circle of 5 cm. radius. Take a point E 8 cm. from its centre O. Through E draw any two secants EBA, EDC. Measure EA, EB and find their product. Do the same for EC, ED. In each case the product should be 39 (sq. cm.). Draw the secant EGF passing through the centre O. Find EG, EF. What is the area of the rectangle contained by EG, EF?

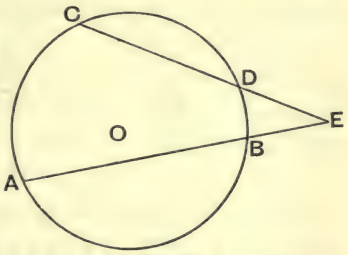


FIG. 268.

8. Repeat the above with a circle of 3 inches radius, the point E being taken 4 inches from its centre. What is the area of the rectangle contained by the whole of any secant, and the part of that secant which lies outside the circle?

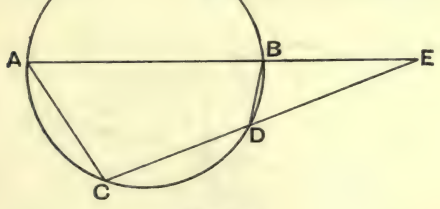


FIG. 269.

9. Repeat for Fig. 269 the instructions given for Fig. 266 (XXIV. 18).

10. When the secant EDC is drawn so as almost

to touch the circle, the points C and D move up very close together. What happens when the secant

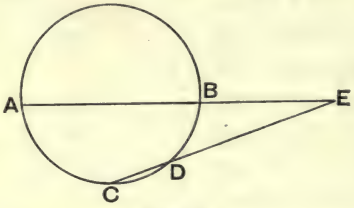


FIG. 270.

becomes a tangent? (XXII. 9.) What does the rectangle $EC \cdot ED$ become?

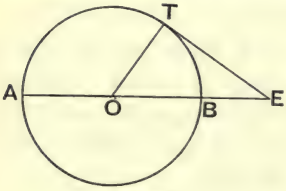


FIG. 271.

11. A point E is taken 7 cm. from the centre of a circle of 4 cm. radius. From E a tangent ET is drawn to the circle. Show by calculation that $ET = 5{\cdot}74\ldots$ cm. and verify your result by making an accurate construction and measuring ET. (Compare Euc. II. 6.)

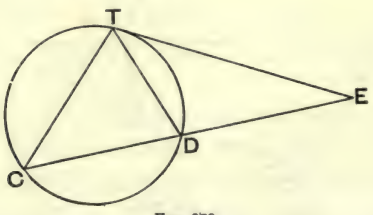


FIG. 272.

12. In Fig. 272, ET is a tangent and EDC a secant. Find out whether the triangles ETD, ECT are similar or not, and which are the equal angles (XXIV. 25).

Write down the proportion for the sides about the angles at E. Compare this with the result obtained in Ex. 10. To which two lines is ET a mean proportional?

13. A chord of a circle 5 cm. long is produced 4 cm. further, and from this further point a tangent is drawn to the circle. Find the length of the tangent.

14. A circle has to be drawn passing through the points C and D and touching the straight line AB

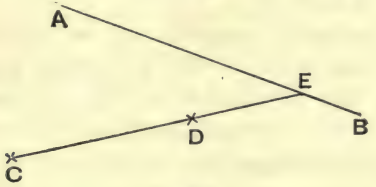


FIG. 273.

(Fig. 273). We have first to find the point of contact. Join CD and produce it to meet AB at E. Find a mean proportional to EC, ED, as shown in Fig. 274,

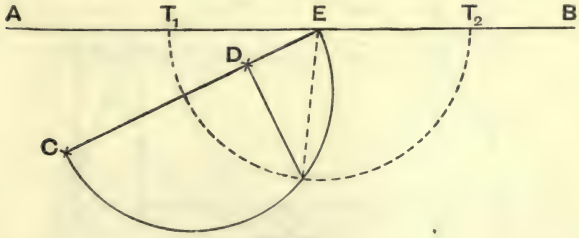


FIG. 274.

and mark off ET equal to this mean proportional. Why is T the point of contact? The centre can

be obtained by drawing the perpendicular to AB at T, and the right bisector of CD.

Show that two circles can be drawn satisfying the required conditions.

Copy this construction and find the centres of the two circles.

15. Draw a straight line AB and take two points C and D distant respectively 2 in. and 1 in. from the straight line, the distance CD being 2 in. Describe two circles each touching AB and passing through the points C and D.

16. A quadratic equation can be solved geometrically by the help of the foregoing properties of the circle. Let us take as an example the equation

$$x^2 - 10x + 16 = 0.$$

This may be written

$$10x - x^2 = 16, \quad \text{or} \quad x(10 - x) = 16.$$

The problem is to divide a line of length 10 into two parts x and $10 - x$, such that the rectangle contained by them may be equal to a square of area 16.

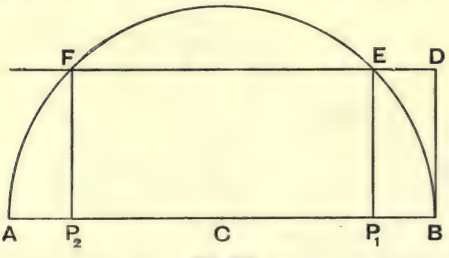


FIG. 275.

Take AB 10 units (centimetres will do). (Fig. 275.) On AB describe a semicircle. Erect BD perpendicular

to AB and of such a length that its square is 16 units. (What length is this?) Through D draw DEF parallel to AB meeting the semicircle in E and F. Draw EP_1, FP_2 perpendicular to AB. Then the number of units of length in AP_1 and in AP_2 gives each of the values of x required. Explain how this follows from the results of Ex. 10, and solve in this way the equation

$$x^2-10x+16=0.$$

17. Similarly solve the equations:

$$x^2-15x+36=0,$$
$$x^2-\ 6x+\ 8=0,$$
$$x^2-\ 5x+\ 5=0,$$
$$3x^2-\ 7x+\ 3=0.$$

18. An equation of the form $x^2-2x-8=0$, where the last term is negative, requires a different construc-

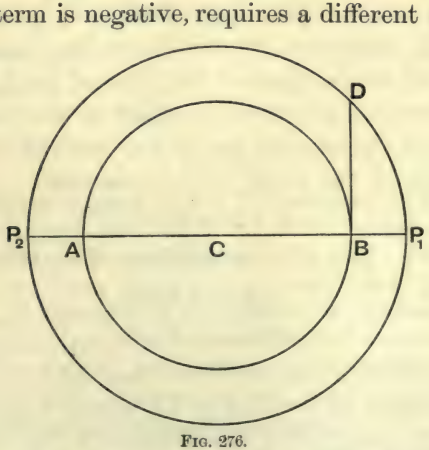


FIG. 276.

tion. It may be written in the form $x(x-2)=8$, and the problem is to produce a line of length 2 to a point

such that the rectangle contained by the whole line and the part produced may be equal to a square of area 8.

Take AB 2 units (Fig. 276). Describe a circle on AB as diameter. At B erect a perpendicular $BD = \sqrt{8}$. With centre C, radius CD, describe a circle, and produce AB both ways to meet it in P_1, P_2. The values of x are given by the lengths AP_1, $-AP_2$. Show how this follows from Ex. 4, and employ this method to solve the above equation.

19. Solve as in Ex. 18 the following equations:

$$x^2 - 6x - 16 = 0,$$
$$x^2 - x - 12 = 0,$$
$$x^2 - 5x - 5 = 0,$$
$$3x^2 - 7x - 3 = 0.$$

20. The equation $x^2 - 6x + 9 = 0$ has equal roots. Solve it by the method of Ex. 16, and point out how the construction shows the equality of the roots.

21. Find, by making use of the method of Ex. 16, the nature of the roots of the equation

$$x^2 - 2x + 4 = 0.$$

22. Give general instructions for solving the equations

$$x^2 - ax + b = 0,$$
$$x^2 - ax - b = 0,$$

similar to those given in Exs. 16 and 18.

The student is referred to Chrystal's "Algebra for Beginners" for a full explanation of this method of solving quadratics.

ADDITIONAL EXERCISES.

XXIX. a.

1. Two straight lines AB and CD cross each other at O. How would you ascertain by measuring the lengths of the segments, whether the four points A, B, C, D lie on a circle or not?

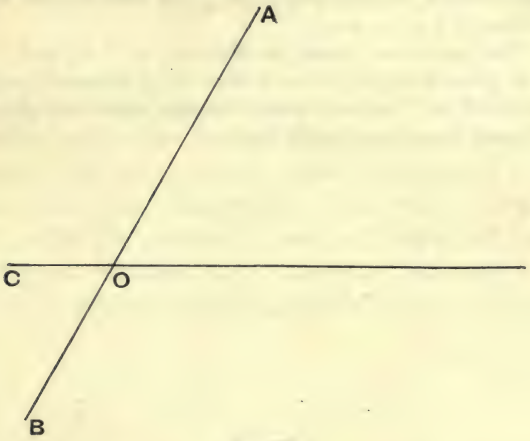


FIG. 277.

In the given diagram (Fig. 277), determine, in centimetres, the distance from O at which the circle through the points A, B, C would cut the line CO produced.

2. If two chords of a circle cut each other, prove that the rectangle contained by the segments of one chord is equal to the rectangle contained by the segments of the other.

Chords of a circle being divided so that the rectangles contained by their segments are each equal to a given rectangle, find the locus of their points of section.

3. A chord of a circle is 3 inches long. It is produced one inch further, and from the end of the produced part a tangent is drawn to the circle. Find its length.

4. Show how to describe a circle passing through two given points and touching a given straight line.

5. Two unlimited straight lines intersect at A; through a given point B draw a straight line so as to make with the former lines an isosceles triangle, having A for vertex. Is there more than one solution?

6. Show how to apply the preceding exercise to the problem of describing a circle to touch two given straight lines and to pass through a given point.

7. Two points are taken at distances of 1 in. and 1·8 in. from a given straight line such that their distance from each other is 1·2 in.; draw a circle through these two points so as to touch the given straight line.

CHAPTER XXX.

SCALES.

1. You have been using hitherto two **scales**, one showing inches and tenths, the other centimetres and tenths. You have now to make for yourself a scale which will show hundredths of an inch. To begin with, make for yourself a scale of inches and tenths, the longest distance to be 5 inches. Draw a line AB exactly 5 inches long (Fig. 278). Divide it into 5

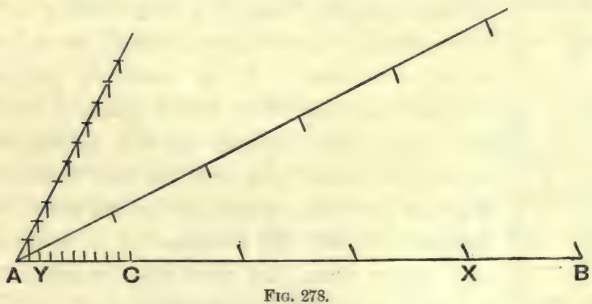


FIG. 278.

equal parts. It is not necessary to show more than the beginning and end of the 5 parallels which you draw. Now to show tenths of an inch, you need only divide the *end inch* AC into ten. Do this by a separate construction as shown in Figure 278. Now

you can take off from your own scale any number of inches and tenths up to 5 inches. For instance, the length XY is 3·8 inches. Draw *from your own scale* lines of the following lengths:

1·2 inches, 2·4 inches, 3·5 inches, 4·6 inches.

Test the accuracy of your work by your wooden scale.

Now you will find it a help to have your own scale numbered. Number all the inches, and the even numbers among the tenths. *Remember that the numbering is to help you in taking distances from your scale. Where does* 0 *come in your numbering?*

2. If you try to divide a tenth of an inch directly into ten parts, you will probably find that the parts cannot be distinguished however sharp your pencil may be. So it is necessary to show the different hundredths of an inch on different lines. Take AB (Fig. 279) a tenth of an inch. Draw ten parallels to AB *at equal distances.* At B draw a perpendicular meeting the tenth parallel at C. Join AC. Now going up the ladder AC from the foot, you will notice that the portions of the parallels intercepted between AC and BC increase by a tenth of AB every time, the first step being

$$\tfrac{1}{10} \times \tfrac{1}{10} = \tfrac{1}{100}.$$

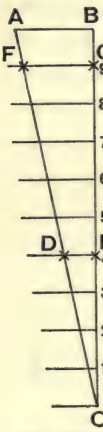


FIG. 279.

The second is two hundredths, and so on. $DE = 0{\cdot}04$ in.: $FG = 0{\cdot}09$ in. $AB = 0{\cdot}10$ in. = a tenth of an inch.

(The triangle FGC is similar to ABC. But

$$CG = \tfrac{9}{10} \text{ of } CB \; ; \; \therefore \; FG = \tfrac{9}{10} \text{ of } AB.)$$

3. Take AB an inch long (Fig. 280.) Divide it into tenths. Rule ten more lines parallel to AB at equal distances, say half a centimetre. Use the 60° set square as in Fig. 58, and slide it 1 cm. along the ruler each time. This gives parallels 0·5 cm. apart (X. 12.) At A and B draw perpendiculars. Join the point C (where the perpendicular from B cuts the last parallel) to D, the first of the **tenth** divisions. Through each of the other **tenth** divisions draw slanting lines (**diagonals**) parallel to CD. You should be able to make out for yourself that the distance between the two points marked *——* in Fig. 280 is 8 tenths + 7 hundredths (= 0·87 in.). Mark in a similar way on your own figure distances of 4 tenths + 5 hundredths, 0·37 in., 0·93 in.

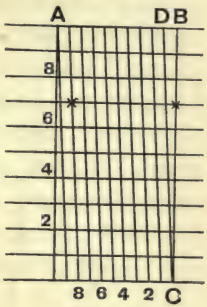


FIG. 280.

4. Fig. 281 shows a **diagonal scale** of inches and hundredths of an inch complete up to 5 inches. The construction lines are shown. Notice that the end perpendiculars are drawn before the ten parallels. Afterwards the other perpendiculars and the nine diagonals are drawn.

What is the distance between the two points marked *——* on the figure?

Draw the figure for yourself and mark on your own scale distances of

0·09 in., 1·5 in., 3·47 in.

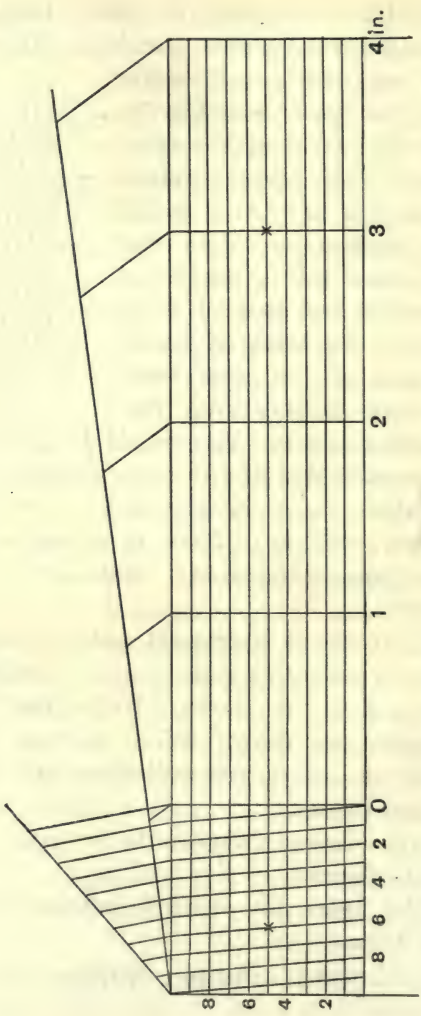


FIG. 281.

5. Portraits and statues are sometimes made *life size*. In these the length of the nose is the same as that of the original, and all the other features are of the same length, *and breadth as well*, as the original. When it is inconvenient to have a life size likeness, it is necessary that all these lengths and breadths should be reduced *in the same ratio*. The same is true of maps and plans. Make a plan of a rectangular room 40 ft. long by 30 ft. broad, all your distances being $\frac{1}{120}$ of the real distance. Find out from your plan the actual distance between two opposite corners of the room.

6. In the plan of the preceding exercise, all distances are $\frac{1}{120}$ of the real distance. The fraction $\frac{1}{120}$ is called the **Representative Fraction** of the plan. Find the Representative Fraction (R.F.) in each of the following.

(i) A plan on a scale of 1 inch to the yard.

(ii) „ „ $\frac{1}{10}$ inch to the foot.

(iii) A map „ 1 inch to the mile.

(iv) The 25 inch ordnance map, in which 25 inches represent 1 mile.

7. A four-sided field $ABCD$ is found to have the following measurements:

$AB = 30$ yards.

$BC = 75$ yards.

$CD = 50$ yards.

$AD = 80$ yards.

Diagonal $AC = 80$ yards.

Make a plan of the field, scale 20 yards to an inch, and find the length of the diagonal *BD*. Can you find in square yards the area of the field? (XIX. 15.)

8. To save the trouble of making a separate calculation for every distance, it is found convenient to make a scale. For instance, for the plan in the preceding exercise, make a scale of yards showing 100 yards. This will be 5 inches long. Divide it into 10 equal parts. Each of these will be 10 yards. Divide the last of these divisions into 10. These will be single yards. The final appearance of your scale should be as shown in Fig. 282. From your own scale make a plan and find the area of the field, comparing your results with what you obtained before. Find the R.F. and write it above the scale in the place indicated.

9. In a map of half an inch to the mile, what is the R.F., and what length on the map will represent 5000 yards?

On a French map, 1 cm. represents a distance of 1 kilometre (1000 metres). Find the R.F. of the map, and make a scale of miles and furlongs for use with the map, showing 6 miles.

10. Fig. 283 shows a scale of feet and inches. Using this scale, make a ground plan of the room you are in, showing the position of doors, windows, fireplace, etc. Of course it will be necessary first to find the actual measurements of the various distances. When these are obtained, draw a rough sketch plan and write in the distances, as in Fig. 284.

Fig. 282.

Fig. 283.

11. Fig. 284 is an inaccurate sketch plan of a room. Draw an accurate plan using the scale in

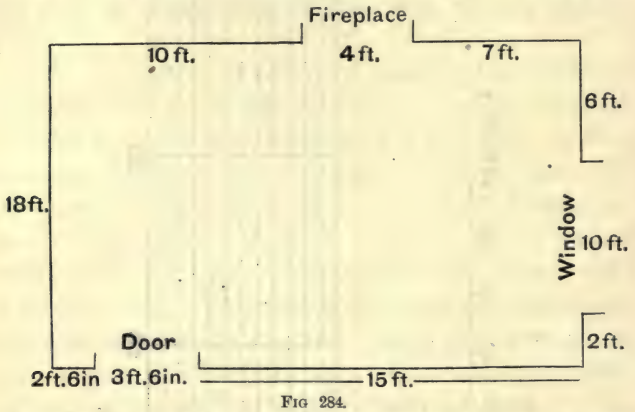


FIG 284.

Fig. 283. Find the distance from the middle of the window to the middle of the doorway.

12. Make a scale of feet and inches, 8 feet to the inch. The inches must be shown diagonally. From your scale draw a plan of the room in Ex. 11, and measure again the same distance. You ought to obtain the same result as before.

13. The Representative Fraction is usually reduced to the form $\frac{1}{\ldots\ldots}$. For instance if the scale were 5 inches to a yard, the R.F., instead of being written $\frac{5}{36}$, would be written $\frac{1}{7 \cdot 2}$.

The distance between two places is known to be three quarters of a mile. The distance between them on a map is measured and found to be 19 inches. What is the R.F. of the map?

Scale of feet and inches.

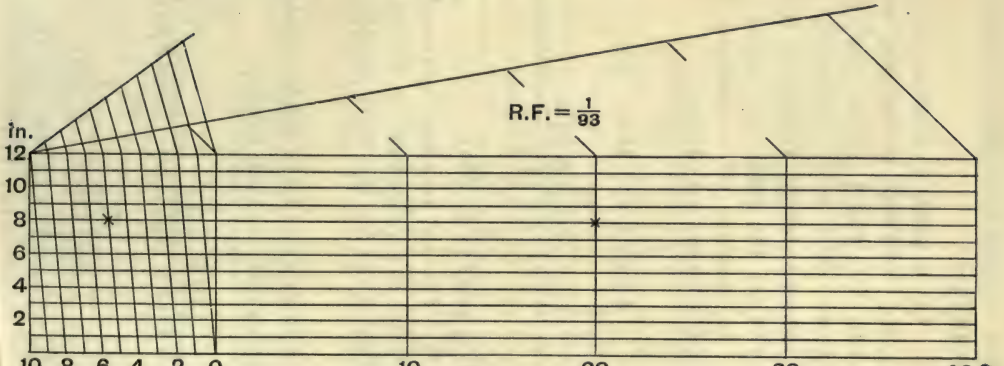


Fig. 285.

The following examples of working scale problems may be found useful:

EXAMPLE I. On a plan, a distance of 37 feet is shown by $4\frac{3}{4}$ in.; construct a diagonal scale representing 50 feet, by which single inches may be measured. Figure your scale properly, and write above it the representative fraction. Indicate, by two small marks on the scale, the points you would take in order to measure off a distance of 27 feet 8 inches. Show all your calculations.

$$\text{R.F.} = \frac{19}{4 \times 37 \times 12} = \frac{19}{1776} = \frac{1}{93\ldots}$$

$$50 \text{ feet occupy } \frac{19 \times \overset{25}{\cancel{50}} \times \cancel{12}}{\underset{2}{\cancel{4}} \times 37 \times \cancel{12}} = 6{\cdot}42 \text{ in.}$$

See preceding page (Fig. 285).

EXAMPLE II. On an Austrian map a distance of $2\frac{1}{2}$ Austrian miles is represented by 1·15 in. Draw a scale of English miles for the map, showing 30 miles. Give R.F.

(1 Austrian mile = 3·312 English miles.)

(1 English mile = 1760 yards.)

$$\text{R.F.} = \frac{1{\cdot}15}{2\frac{1}{2} \times 3{\cdot}3312 \times 1760 \times 36} = \frac{\overset{23}{\cancel{1{\cdot}15}}}{\underset{50}{\cancel{250}} \times 3{\cdot}3312 \times 63360}$$

$$= \frac{23}{10553241{\cdot}6} = \frac{1}{458837}.$$

$$30 \text{ miles occupy } \frac{23 \times 30 \times \cancel{63360}}{50 \times 3{\cdot}3312 \times \cancel{63360}} = \frac{69}{16{\cdot}656} = 4{\cdot}14 \text{ in.}$$

Scale of miles.

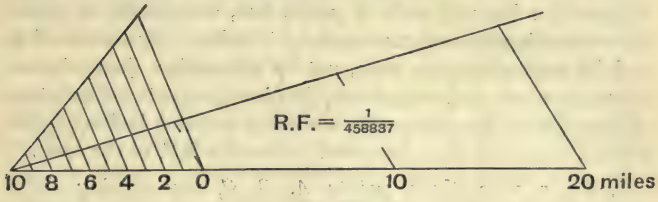


FIG. 286.

ADDITIONAL EXERCISES.

XXX. a.

1. Draw an area of 10 square inches. Supposing this represents an area of one acre, find the representative fraction.

2. Find the error in reckoning that the area of the country covered by a halfpenny (diameter one inch) on an ordnance map, whose scale is one inch to the mile, would be 500 acres.

3. Draw a plain scale of yards, R.F. $\frac{1}{6\,\frac{1}{60}}$, to show 100 yards; and a comparative scale of metres, given 1 metre = 39·37 in.

4. The latitude of London is 51° N., and the radius of the Earth 4000 miles. How far is London from the Equator measured along the Earth's surface, and how far from the Earth's axis?

5. Construct a plain scale of yards (to read to 100 yards) for a road sketch, which has been executed on horseback, the scale employed being $\frac{1}{4}$ inch for every 400 times the rider rose in his saddle at a trot; the pace of the horse at a trot was such that the rider would rise in the saddle on an average 34 times in 100 yards. Give the R.F. of the scale.

6. A distance of 8 miles 3 furlongs is represented by 2·4 inches. Draw a scale to show 20 miles and single miles.

7. Construct a scale of kilometres for an English Ordnance map of 1 inch to the mile.

(1 metre = 3·28 feet.)

8. Two poles are fixed in a level piece of ground so that their centre lines are vertical and 4 yards 2 feet 2 inches apart. Construct a diagonal scale of yards, feet, and inches, in which the above distance is represented by 5 inches. By construction and by employing your scale, *not* by calculation, determine to the nearest inch, and write down, the distance apart of the centres of the tops of the poles, one pole being 2 feet 9 inches higher above the ground than the other. Give the R.F. of the scale.

9. On a map, three points A, B, C, representing three mountain tops, lie in order in a straight line, and the heights are stated to be 2650, 2451, and 2315 feet respectively. Given that $AB = 3\frac{1}{4}$ inches and $BC = 2\frac{5}{8}$ inches, determine whether the top of C can be seen from the top of A.

10. Make a plain scale of miles and furlongs, to show 6 miles, for a map with representative fraction $\frac{1}{75000}$.

11. Find the representative fraction for a plain scale on which 207 yards is measured by 2·3 inches: and make a scale to measure up to 500 yards.

12. Construct a plan of a 5-sided field $ABCDE$ from the following measurements: $AB = 270$ yds., $BC = 320$, $CD = 125$, $DE = 400$, $EA = 210$, $AC = 480$, $BD = 365$ yds. Scale, which should be drawn and figured, 120 yards to an inch. Reduce the figure to a triangle and find its area.

13. Construct a pentagonal figure $ABCDE$ with the following measurements: $AD = 130$ yards; the perpendiculars on AD from B, C, E, are 55 yards, 75 yards, 60 yards; the feet of these perpendiculars are distant 20 yards, 100 yards, and 80 yards from A respectively. Scale, 30 yards to an inch.

Compute the area of the figure.

14. A man walks 275 m. from A to B, turns at right angles to the right and walks 325 m. to C, then turns to the right through an angle of 105° and walks 300 m. to D.

Draw a plain scale of metres R.F. $\frac{1}{4000}$, and thence plan the walk, constructing the angles. Find the area of $ABCD$.

15. Draw the figure $ABCDE$, having given that $AB = 28$ ft.,

$BC=16$ ft., $CD=24$ ft., $DE=22$ ft., $AE=16$ ft., $AC=34$ ft., $AD=32$ ft.

Scale, which must be drawn, $\frac{1}{132}$.

16. Draw a plain scale of miles and furlongs, showing 5 miles, to suit a map on which a distance of 3 miles is represented by 3·6 in. Find the R.F.

17. The R.F. of a plan is $\frac{1}{14}$. Draw a plain scale of feet and inches to suit the plan, showing 6 feet.

18. A rectilinear field $ABCDE$ has $AB=44$ yards, $BC=CD=28$ yards, $DE=20$ yards, $EA=31$ yards, $BD=40$ yards, and the angle $ABD=60°$.

Draw a plan of the field to a scale of 20 yards to the inch, and find its area. [The scale need not be drawn.]

19. It is required to find the range of two of an enemy's forts P and Q, and their distance from each other. To do this, a base line AB of 1500 yards is taken and the following angular measurements are made :

$$BAP=95°,\quad ABP=40°,$$
$$ABQ=115°,\quad QAB=20°.$$

Draw a plan to the scale 3 inches to a mile.

20. Construct a figure $ABCDE$ having given $AB=100$ yards, $\angle ABC=135°$, $\angle BAE=120°$, $AE=75$ yards, $AC=160$ yards, $ED=90$ yards, and $DC=150$ yards.

Reduce the figure to a triangle, draw a rectangle equal to the triangle, and a square equal to the rectangle.

Make a scale of 40 yards to the inch.

21. On a map 27 miles 2 furlongs are represented by 7 inches. Find the R.F. and draw a diagonal scale of miles and furlongs showing 20 miles.

22. A plan drawn to a scale the R.F. of which is $\frac{1}{11}$ is to be copied to a scale half as large again as the original. Construct a scale of feet and inches for a new plan (showing 4 feet) and write above it its R.F.

23. On a French map the distance between two points known to be exactly one kilometre (3280·9 feet) apart measures $\frac{7}{10}$ths of an inch, construct a comparative scale of miles and furlongs for the map and give its R.F.

24. On a map 67·5 miles are represented by 9·3 inches. Draw a scale of miles for the map showing 30 miles, and show furlongs by the diagonal method. Mark in the scale a distance of 17 miles 3 furlongs.

25. Draw a diagonal scale of feet and inches, showing 40 feet, to suit a plan whose R.F. is $\frac{1}{75}$. Indicate by two small marks on the scale a distance of 24 feet 7 inches.

26. The distance between two points is known to be 440 yards, and measures on an English plan exactly 4 inches. Draw a diagonal scale of metres for the plan. Show all the calculations, and write above it its representative fraction.

27. A plan is on a scale of 1 inch to 53 yards 1 foot. Draw a scale to measure 300 yards, and to measure single yards diagonally. Print R.F. Show on the scale a length of 173 yards.

By the side of this place a comparative scale of metres. 1 kilometre = 0·62 miles.

28. If 100 feet are to be represented by 13·4 in., draw a scale of feet showing inches diagonally. Draw two lines AB and AC, 27 ft. 5 in. and 32 ft. 7 in. respectively containing an angle of 45°. Measure BC and calculate the area of ABC to the nearest square foot.

29. Draw a diagonal scale of yards, feet and inches for a map on which 2 yards 1 foot is represented by 1·75 in., showing 6 yards. State R.F.

30. A plan is 0·875 mile to 1 inch. Lay down a scale of miles for it, marking furlongs and poles diagonally.

A portion of this plan is to be enlarged, so that any given distance shall be six times as long. Draw a scale of yards, up to 1000, for it.

31. A train is running in a straight line at 12 miles an hour. The door of one of the carriages swings open, so that each point of the door describes with uniform velocity a quarter of a circle about a hinge, and immediately closes again at the same rate, the whole time of opening and closing being half a second. The width of the door is 27 inches. Draw the curve traced out by a point on the edge of the door.

Scale (which need not be constructed) $\frac{1}{2}$ inch = 1 foot.

32. Suppose a line, 13 inches long, denotes on a map the distance between two towns, which are 85 miles apart, lay down a scale of miles up to 40, showing furlongs diagonally. Set off a distance 22 miles 5 furlongs.

33. The sides of a triangle are 10 chains, 8 chains and 12 chains. Find the acreage of the field, and the perpendicular distance of its longest side from the opposite corner.

34. On a map 4·8 kilometres are represented by 1·5 decimetres. Draw a diagonal scale to read decimetres. Show 5 kilometres. Give the R.F., and from the scale draw a line to represent 3·87 kilometres.

CHAPTER XXXI.

GRAPHS.

1. A and B are two places 10 miles apart. A man starts from A and walks 1 mile towards B. How far is he from B? He then walks 3 miles in the opposite direction. How far is he now from B?

2. You can see from Ex. 1 that distances have to be added or subtracted according to the direction, or **sense**, in which they are measured. In Fig. 287 B is

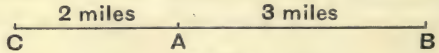


FIG. 287.

3 miles East of A and C is 2 miles West of A. We can express this by saying $AB = +3$ miles,

$AC = -2$ miles.

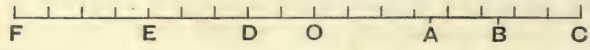


FIG. 288.

Similarly write down the distances OA, OB, OC, OD, OE, OF. (Fig. 288).

3. Fig. 289 shows the linesof latitude and longitude of a portion of the earth's surface near the equator and near the meridian of Greenwich, each of

which is taken as 0° by English geographers. Observe that the place A is in longitude 1° E, latitude 2° N.

What are the latitude and longitude of B? of C?

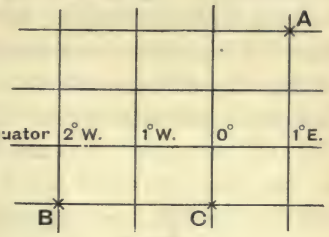


FIG. 289.

4. If East is +, then West is −. Similarly if North is +, then South is −.

Thus, in Fig. 290, the point P, which is 3 spaces East, and 2 North, may be denoted by the numbers

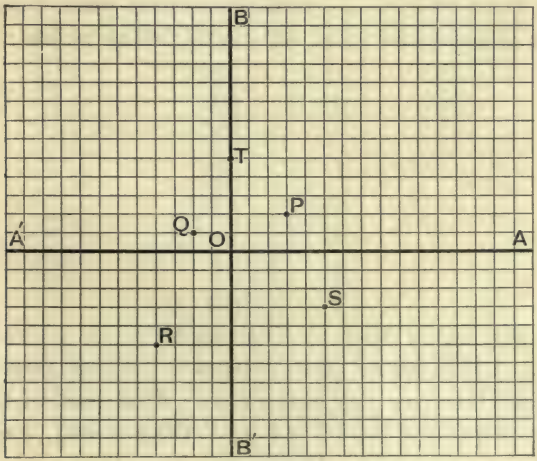


FIG 290.

(3, 2) in brackets. Similarly Q is denoted by the numbers $(-2, 1)$.

How would you denote the points R, S, T? The

spaces can be any distances you please from millimetres to geographical degrees, and the distance along the line OA is called the **abscissa** instead of the longitude, while the distance above or below the line OA is called the **ordinate** instead of the latitude. The lines OA, OB are called the **axes.** If the position of a point is uncertain, we must use letters instead of numbers for its abscissa and ordinate, and the letters x and y are always used for this purpose. Hence, the line OA along which the abscissa is measured is called the **axis of** x, OB **the axis of** y; and x and y are called the **coordinates** of the point.

5. Take two of the darker lines on your squared paper as axes, and mark off the following points:

x	y
3	4
2	5
1	6
0	7
-1	8
-2	9
-3	10

You will notice that all the points lie in a straight line, and that for all them $x+y=7$. Where does the straight line cut the axis of x? Find out whether other points for which $x+y=7$ (like $x=4\frac{1}{2}$, $y=2\frac{1}{2}$) all lie on this straight line or not.

This straight line may be called

(i) the **locus** of all points of which the sum of the abscissa and ordinate is 7.

(ii) the **graph** of the equation $x+y=7$.

6. To draw the graph of an equation, it is necessary to obtain a number of points on the graph, by giving various values of x and finding the corresponding value of y. For instance, to find the graph of $x-y=2$. The equation may be written $y=x-2$. Tabulate the values thus:

x	-3	-2	-1	0	1	2	3	4
y	-5	-4	-3	-2	-1	0	1	2

These points are all shown in Fig. 291. By joining

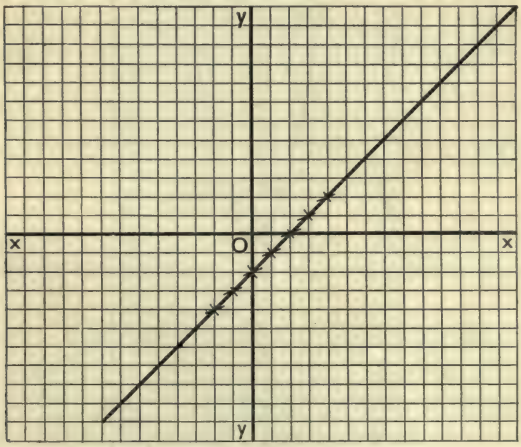


FIG. 291.

them we obtain the graph. The graph is not always a straight line. For instance the graph of

$$y=x^2-x+2$$

may be obtained thus

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$y(=x^2-x-2)$	4	0	-2	-2	0	4	10

The graph is obtained by joining the points by a curve drawn freehand (Fig. 292).

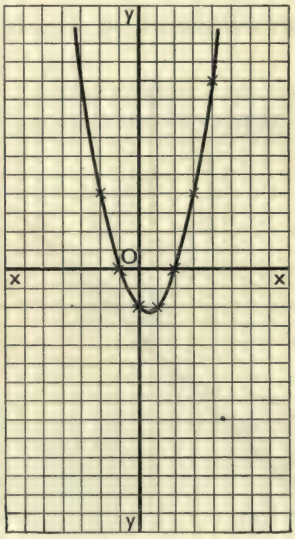


FIG. 292

Draw the graphs of the following:

(i) $y=x$.

(ii) $x+y=0$.

(iii) $y = 2x + 3$.

(iv) $2y = x + 1$.

(v) $3x + 4y = 7$.

(vi) $y = x^2$.

7. Squared paper and curve-tracing may be made useful in many ways. For instance, take as abscissae the time reckoned in hours, for ordinates the temperature in degrees, and we obtain a curve showing the variation of temperature during a day (Fig. 293).

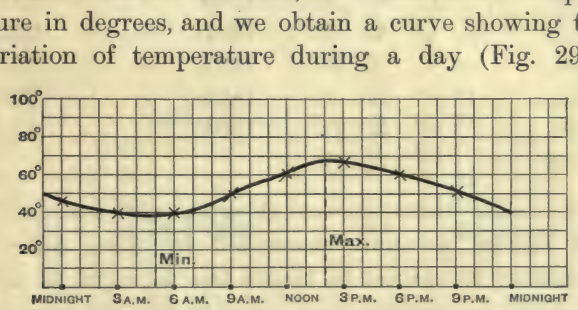


FIG. 293.

Notice that probably the **maximum** temperature came between noon and 3 P.M., the minimum between 3 and 6 A.M.

Trace a line showing the changes in the height of the barometer as recorded in the following table for a week in 1902. The various points on the curve may be joined by straight lines, instead of a curved line being drawn.

Sept. 1,	- -	29·88	Sept. 5, - - 30·03
„ 2,	- -	29·79	„ 6, - - 30·07
„ 3,	- -	29·80	„ 7, - - 30·20
„ 4,	- -	29·95	

8. Draw a curve showing the variations in the price of wheat in the years 1880-1894, from the following table:

Year.	Price per quarter. s.	d.	Year.	Price per quarter. s.	d.
1880, - -	45	7	1888, - -	31	6
1881, - -	44	8	1889, - -	30	1
1882, - -	46	4	1890, - -	31	5
1883, - -	41	9	1891, - -	35	9
1884, - -	37	10	1892, - -	33	8
1885, - -	33	0	1893, - -	26	7
1886, - -	30	11	1894, - -	21	11
1887, - -	32	10			

9. Draw three curves showing the increase in the annual premium demanded by three different insurance companies to insure the payment of £100 at death to insurers who begin to pay the premium at the age specified, and determine the probable premium in each company for the ages 20 and 42.

	Age 25.	Age 30.	Age 35.	Age 40.	Age 45.	Age 50.	Age 55.
Office No. i.	£2 14 0	£3 0 4	£3 8 6	£3 18 10	£4 12 0	£5 8 4	£6 9 4
Office No. ii.	£1 18 0	£2 1 6	£2 6 10	£2 14 9	£3 5 9	£4 1 7	£5 1 11
Office No. iii.	£1 17 6	£2 4 0	£2 12 2	£3 2 8	£3 16 8	£4 15 4	£6 0 8

10. Draw a curve showing the corrections necessary to obtain the true time from a sundial. If the sundial is 4 min. after the clock, then 4 min. has to be added to the time given by the sundial to obtain the true time. Therefore the times "after clock"

should be marked above the line of ordinates, those "before clock" below the same line. On what days is the sundial correct?

	min.	sec.	
Jan. 1.	3	39	after clock.
„ 15.	9	33	„
„ 29.	13	19	„
Feb. 13.	14	26	„
„ 27.	12	58	„
Mar. 13.	9	26	„
„ 27.	5	16	„
Apr. 10.	1	9	„
„ 24.	2	2	before clock.
May 8.	3	42	„
„ 22.	3	30	„
June 5.	1	40	„
„ 19.	1	12	after clock.
July 3.	4	3	„

	min.	sec.	
July 17.	5	57	after clock.
„ 31.	6	7	„
Aug. 14.	4	23	„
„ 28.	0	55	„
Sept. 11.	3	40	before clock.
„ 25.	8	34	„
Oct. 9.	12	53	„
„ 23.	15	41	„
Nov. 6.	16	14	„
„ 20.	14	5	„
Dec. 4.	9	21	„
„ 18.	2	52	„
„ 31.	3	32	after clock.

11. Find from the tables the values of $\cos x$ when $x = 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°$. Draw a curve on squared paper, showing how $\cos x$ varies as x increases from 0° to 90°.

Find from the curve the values of cos 25° and cos 45° (explaining how you find them), and verify your values by means of the tables.

12. As in the previous Exercise draw a curve showing how sin x varies as x increases from 0° to 90°. Find from the curve the values of sin 63° and sin 35°.

13. Draw in a single figure curves showing the variation in the circular measure (radian measure), the sine and the tangent of an angle as the angle increases from 0° to 90°.

14. Taking rectangular axes, plot off the points $(-1, 2)$ and $(3, 4)$, and draw the line represented by

$$2x - y - 3 = 0.$$

Find the coordinates of the point on the given straight line which is equidistant from the given points.

15. Draw two curves showing the decrease in crime and the increase of school attendance in Great Britain, plotting them from the following tables:

CRIMINAL CONVICTIONS.			EDUCATION, GREAT BRITAIN.
YEAR.	ENGLAND AND WALES.	SCOTLAND.	AVERAGE SCHOOL ATTENDANCE.
1874	11,509	2,231	1,985,394
1875	10,954	2,205	2,175,522
1876	12,195	2,051	2,340,277
1877	11,942	2,009	2,511,096
1878	12,473	2,273	2,782,454
1879	12,525	2,091	2,980,104
1880	11,214	2,046	3,155,534
1881	11,353	1,832	3,273,501
1882	11,699	1,944	3,436,416
1883	11,347	1,914	3,560,351
1884	11,134	2,077	3,721,366
1885	10,500	1,956	3,826,980
1886	10,686	1,838	3,915,315
1887	10,338	1,843	4,019,116
1888	10,561	1,853	4,111,206
1889	9,348	1,723	4,185,725
1890	9,242	1,928	4,230,607
1891	9,055	1,822	4,288,321
1892	9,607	1,778	4,409,452
1893	9,797	1,903	4,642,881
1894	9,634	1,937	4,793,276

EXAMINATION PAPERS.

NAVY. MARCH, 1900.

GEOMETRICAL DRAWING.

[N.B.—*Each figure should be neatly drawn in clear, fine pencil lines on the same page as the question, and if time allows, it should be inked in.*

No writing whatever is allowed on the blotting paper.

The solutions must be strictly geometrical, and particular care should be taken to show all the necessary lines of construction.]

1. Draw two straight lines meeting at an angle of $67\frac{1}{2}°$. Obtain the angle by construction.

2. Describe a circle having its centre in the line *ab*, and passing through the points *c*, *d*.

3. Draw a straight line 3·8 inches long, and divide it by construction into 5 equal parts.

4. Construct a triangle having two sides 2·5 and 1·8 inches long, and the angle contained between them (which may be protracted) 38°.

Construct another triangle, similar to the first, and of twice the area.

5. Describe a circle of $1\frac{1}{2}$ inch radius, and describe within it three equal circles, touching each other and the outer circle. Ink in the outer circle with a fine dashed line, and the inner circles with thick continuous lines.

6. Construct a diagonal scale to show feet and inches. R.F. $= \frac{1}{120}$. Show by two small dots on the scale a length of 43 ft. 8 inches.

NAVY. NOV., 1901.

GEOMETRICAL DRAWING.

[N.B.—*The methods used must be geometrical, except where other methods are expressly prescribed in the Question Paper. In the absence of special instructions, set squares, parallel rulers and protractors may be used. No credit will be given for trial methods.*]

1. Make a triangle with sides 5, 6, and 7 inches long. Measure the angles with a protractor and write down their values.

State your construction.

2. A man walks 1 mile due *S.* from *O* to *A*, then successively $2\frac{1}{4}$ miles *E.S.E.* from *A* to *B*, $3\frac{1}{2}$ miles *N.E.* from *B* to *C*, $3\frac{1}{2}$ miles due west from *C* to *D*, and, finally, straight from *D* to his starting point *O*.

Draw a plan of his walk, taking 1 inch to represent 1 mile. Give the length of *DO* in miles.

(You may either construct or protract the angles.)

3. Draw straight lines through *P* and *Q* to touch the given circle. State your construction.

4. A map is constructed on a scale of 1 inch to each mile. Make a diagonal scale for the map to show miles, tenths, and hundredths.

Indicate by two small dots on your scale a distance of 3·47 miles.

To obtain full marks the scale must be inked in.

5. Draw the curve traced by the point *P* when the given circle rolls on the given line. State your construction.

NAVY. MARCH, 1902.

GEOMETRICAL DRAWING.

1. Make a triangle having two sides respectively 13·2 and 10·7 centimetres long, and containing an angle of 60°.

Measure the third side and the other two angles.

2. From two points *A* and *B* on a line of earthworks, the bearings of a Boer gun are respectively N. 16° E. (that is 16 degrees East of North) and due North. If *B* is East-South-East of *A* and 2000 yards from *A*, find the distance of the gun from *A*.

Scale (which need not be drawn) 1000 yards to an inch.

3. Draw a line 3·5 inches long. Bisect it. On each half as diameter describe a circle. Describe an equal circle to touch both of these circles.

State your construction.

4. Draw a plain scale of feet and inches. R.F. $\frac{1}{25}$.

For full marks this scale must be inked in.

5. From the point *O* on the given circle, diameter *OB*, draw lines *OA* to the circumference, and produce *OA* both ways. From *A* mark off in each direction from *A* a length *AD* equal to the diameter *OB*. Draw the curve traced by the point *D* as *A* moves round the circumference of the circle. (Do not ink this in.)

NAVY. MARCH, 1902.

GEOMETRY.

(Euclid, Books I., II., and III., with easy deductions.)

The demonstrations need not be those of Euclid.

1. Two points, A and B, are marked on the edge of a thin piece of cardboard. The cardboard is placed on a sheet of paper, and the edge from A to B traced on the paper. The cardboard is then turned over, and laid again on the paper with A and B at the same points as before. In what circumstances will the edge then coincide with the line drawn on the paper?

How would you test whether the edge of a flat ruler is straight?

2. Suppose the triangles PQR and XYZ cut out of cardboard, so that $PQ=XY$, and $PR=XZ$, and the angle P=the angle X. Suppose P placed on X, and the side PQ laid along XY, and then the two surfaces brought together. How do you know (1) that Q falls on Y, (2) that PR falls on XZ, and (3) that R falls on Z?

3. A circle of radius 1 inch rolls round on a fixed circle of radius 2 inches, the two circles being all the time in the same plane. Find the locus of the centre of the rolling circle.

4. Draw, with your instruments, five triangles having sides of

(a) 2 inches, 4 inches, 5 inches.
(b) 3 inches, 3 inches, 5 inches.
(c) 3 inches, 4 inches, 5 inches.
(d) 4 inches, 4 inches, 4 inches.
(e) 4 inches, 4 inches, 5 inches.

Write under each whether it is equilateral, isosceles, or scalene; and whether it is acute-angled, right-angled, or obtuse-angled.

5. Prove that parallelograms on the same base and between the same parallels are equal to one another.

Take a straight line 5 centimetres long. Describe a parallelogram on it, having one angle equal to one-half of a right angle, and an area of 40 square centimetres. Measure carefully one of its longest sides, and write down your measure.

6. If I walk 10 kilometres due eastwards, and then 4 kilometres due northwards, how far am I finally from my starting point? Give the distance to the nearest kilometre. (You may either calculate the distance, or measure it from a figure carefully drawn to scale.)

7. Show (with proof) how to divide a given straight line into two parts such that the rectangle contained by the whole line and one of the parts shall equal the square on the other part.

If the given line be 70 inches long, calculate to the nearest inch the length of the greater part.

8. Show (with proof) how to draw a straight line from a point outside a given circle to touch it.

9. Draw a circle of radius 3 inches, and in it place chords of lengths 1, 2, and 4 inches. Ascertain from your figure which of these chords is nearest to the centre and which furthest from it.

State a more general proposition concerning the lengths of chords and their distances from the centre.

Two chords of a circle are respectively $\frac{3}{5}$ and $\frac{4}{5}$ of its radius distant from its centre. Find how many times the shorter chord is contained in three times the longer chord.

COLLEGE OF PRECEPTORS.

LOWER FORMS EXAMINATION.—Midsummer, 1902.

1. Given a straight line of indefinite length and a point, not in it nor in it produced, draw a line through the point parallel to the line.

2. Describe a right-angled triangle such that the side opposite to the right angle is 2 inches and one of the other sides is 1 inch.

3. Given a straight line AB of unknown length and a point C, not in this straight line nor in it produced, give the full construction for drawing a straight line from C equal to AB in length and perpendicular to it.

4. Describe a triangle with sides 5, 6, 7 half-inches respectively, and from each vertex drop a perpendicular on the opposite side.

5. A village A is 5 miles from another village B and 4 miles from a third C, while B is 6 miles from C. Draw a map, stating what scale you choose, and determine the position and distance from A of a church which is at the same distance from each of the three villages.

6. Describe a parallelogram having one side 1 inch in length and one angle equal to that of an equilateral triangle, and having its area equal to that of a square whose side is 2 inches.

7. Describe a triangle with sides 2, 3, 4 half-inches. Produce all three sides, and find, by construction, the centres of the inscribed, circumscribed, and escribed circles.

8. A diameter of a circle is bisected, and on each of its halves as diameter, a smaller circle is described. Construct the figure, and draw tangents (i) to touch both the larger and one of the smaller circles, and (ii) to touch both the smaller circles.

COLLEGE OF PRECEPTORS.

LOWER FORMS EXAMINATION.—Christmas, 1902.

1. Describe a triangle with its sides equal to three given lines of unequal lengths.

Show by figures that a triangle cannot be described having its sides equal to 5, 3, and 2 inches respectively.

2. Describe a triangle with angles one-half of a right angle and two-thirds of a right angle respectively, and the adjacent side 3 inches.

Show, by construction, that an isosceles triangle cannot be described with the above side and angles.

3. Describe a rectangle with side 2 inches so as to be equal in area to an equilateral triangle whose side is also 2 inches.

4. Draw, with constructions, a plan of a cricket-field from the following data, stating the scale you use:—distance between wickets, 22 yards; places of point and square-leg, each 10 yards from the line of wickets on opposite sides in a line at right angles to it drawn through the batsman's wicket; long-off and long-on, each 30 yards from that wicket, on opposite sides in lines making a third of a right angle with the line of wickets; back-stop, 15 yards behind the batsman's wicket.

5. Describe a square of side 2 inches, and inscribe in it a circle of radius 1 inch; describe circles on two opposite sides as diameters; and show by your construction that each of a certain pair of lines touches all three circles.

6. A man has twenty trees, and wishes to place them at equal distances from one another in the boundary of a square field of given side. Show, by construction, where they must be placed.

7. Inscribe an equilateral six-sided figure in a circle whose radius is 1 inch. Draw tangents at its alternate angular points (*i.e.* the first, third, and fifth corners of the figure), and show that these three tangents form the sides of an equilateral triangle. Measure the sides of that triangle.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

* From "Mathematical Tables for Ready Reference," compiled by F. Castle (Macmillan).

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
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99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TRIGONOMETRICAL RATIOS.

Angle.	Radians.	Chords.	Sine.	Tangent.	Cotangent.	Cosine.			
0°	0	0	0	0	∞	1	1·414	1·5708	**90°**
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	**89**
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	**88**
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	**87**
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5	0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	**85**
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	**84**
7	·1222	·122	1219	·1228	8·1443	·9925	1·325	1·4486	**83**
8	·1396	·139	1392	·1405	7·1154	·9903	1·312	1·4312	**82**
9	·1571	·157	1564	·1584	6·3138	·9877	1·299	1·4137	**81**
10	·1745	·174	1736	·1763	5·6713	·9848	1·286	1·3963	**80**
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16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	**74**
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	**73**
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	**72**
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43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	**47**
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	**46**
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	**45**
			Cosine.	Cotangent.	Tangent.	Sine.	Chords.	Radians.	Angle.

ANSWERS TO ADDITIONAL EXERCISES.

II. a. 6. 123; 4060 cm. 9. 75·022 kilo.

V. a. 2. 1·53 in. 3. 2·64 miles.
4. 1·5 in. 7. 53°; 8 ft.; 6 ft.

V. b. 1. 4·59 in. 2. 19·1 yds.
3. 4 miles. 5. 14, 11·67, 12·8.
6. 25 ft. 7. 16 ft.
9. 6 miles. 10. 54·5 yds.
11. Each = 27·2 yds. 12. 1900 yds.
13. 31 yds.; 18 yds. 14. 30·06 yds.
15. 5·75 miles. 16. 51·94 yds.; 61·94 yds.
17. 4 miles. 18. 70·4 ft.; 45·6 ft.
19. 2732 yds.; 1932 yds. 20. 1 rt. angle; 588 miles.
21. 41·6 ft. 22. 32·3 ft.
23. 5·3 miles. 24. 86·6 ft.
25. 57 ft.; 48 ft.; 88 ft. 26. 1 mile.
27. 80 yds.

VI. a. 1. 2·3 in.; 50·2°. 2. 4·96 in.
4. 14·4 yds. 5. 13·2°.

VII. a. 16. 4400. 17. 42 yds.
18. £3268. 15s. 0d.; 6636 sq. yds.
19. 160 ft. 20. $8\frac{1}{2}$ pieces.
21. £1270. 10s. 0d. 22. 110 yds.
23. 9 min. 8 sec. 24. 2.
25. 1·015538 sq. m.; ·004392 sq. m.

VIII. a. 1. 40000 c. f.; 3600 sq. f.
2. 24000 c. f.; 2800 sq. f.
3. 60 c. in.; 94 sq. in.
5. 27 in. 6. 5460.
7. (1) 2058; (2) £4. 18s. 0d.
8. 247500. 9. 2·207 ....
10. 67200 c. yds.; 50625.
11. 36·8 miles an hour.
12. 3375 cub. in. 13. £2. 13s. 1d.
14. 259285. 15. 2640.
16. 7 : 3. 17. 4·775 ft.
18. 5. 19. 8·55 cub. in.

XII. a. 2. 1·6 in.; 40°. 3. 1·63 in.

XV. a. 8. 35·8 mm. 10. 63 mm.; 82°; 38°.
11. 3·45 cm. 12. 2·14 in., 2·29 in.
13. 2·84 in. 15. 131 yds.; 45°.
17. 68 yds. 22. 45°, 85°.
23. 51°, 59°, 70°. 26. 12·2 miles.

XVI. a. 2. 3·8 in. 6. 2·73 in.

XIX. a. 3. 3 ft. 2 in. 5. 33·4°.
6. 6·93 in.; 8 in. 8. 324 sq. yds.
9. 12·2. 11. 91 mm.; 56 mm.
12. 72 mm.; 135 mm. 15. 39·9.
16. 15·6 sq. in. 19. 3·9.
20. A little more than 2. 22. 5024.

XX. a. 3. 36 ft. 0·7 in. 4. 12 in.
5. 14 in. 6. 76 miles.

XXII. a. 6. 83°. 7. 13 ft. 2 in.

XXVI. a. 1. 58·37 mm.; 48·98 mm.; ratio = 1·19.
3. 3·2; 0·78. 4. 3·12; 2·36.
8. 1·8; 2·24.

XXVII. a. 11. 1·86 sq. in. 21. 19·9 sq. in.

XXVIII. a. **1.** ratio = 1 : 2·41. **2.** 19 tons.
4. 78·540, 12·566. **5.** $2 : \sqrt{3} = 1 : 0{\cdot}866$.
6. 189 sq. in.; 198 c. in. **8.** 90.
9. 6109 yds. **10.** 453 sq. in.; 528 sq. in.
11. 532 ounces. **12.** 5026·56 c.f.; 2613·81 sq. f.
13. volume $\propto$ (depth)^2 × (speed of flow); 3456 cub. m.
15. (*a*) $62\frac{6}{7}$ sq. in. (*b*) $100\frac{4}{7}$ sq. ft. (*c*) $125\frac{5}{7}$ ft.
16. 151 c. i.; 113 c. i.; 2·08 in.
18. 972. **19.** 3,466,145 c. f.
20. 255·7 yds.; 337541 c. yds.
21. 314 c. i. Curved surface = 408·2 sq. in.
22. 2048. **23.** 1·62 in.
24. 5917 **k**. **25.** 4·43 kilometres.

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